

Effects of Lightest Neutrino Mass in Leptogenesis

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Abstract

The effects of the lightest neutrino mass in “flavoured” leptogenesis are investigated in the case when the CP-violation necessary for the generation of the baryon asymmetry of the Universe is due exclusively to the Dirac and/or Majorana phases in the neutrino mixing matrix U . The type I see-saw scenario with three heavy right-handed Majorana neutrinos having hierarchical spectrum is considered. The “orthogonal” parametrisation of the matrix of neutrino Yukawa couplings, which involves a complex orthogonal matrix R , is employed. Results for light neutrino mass spectrum with normal and inverted ordering (hierarchy) are obtained. It is shown, in particular, that if the matrix R is real and CP-conserving and the lightest neutrino mass m_3 in the case of inverted hierarchical spectrum lies the interval $5 \times 10^{-4} \text{ eV} \lesssim m_3 \lesssim 7 \times 10^{-3} \text{ eV}$, the predicted baryon asymmetry can be larger by a factor of ~ 100 than the asymmetry corresponding to negligible $m_3 \cong 0$. As consequence, we can have successful thermal leptogenesis for $5 \times 10^{-6} \text{ eV} \lesssim m_3 \lesssim 5 \times 10^{-2} \text{ eV}$ even if R is real and the only source of CP-violation in leptogenesis is the Majorana and/or Dirac phase(s) in U .

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1 Introduction

In the present article we continue to investigate the possible connection between leptogenesis [1, 2] (see also, e.g. [3, 4]) and the low energy CP-violation in the lepton (neutrino) sector (for earlier discussions see, e.g. [5, 6, 7, 8] and the references quoted therein). It was shown recently [9] that the CP-violation necessary for the generation of the observed baryon asymmetry of the Universe in the thermal leptogenesis scenario can be due exclusively to the Dirac and/or Majorana CP-violating phases in the Pontecorvo-Maki-Nakagawa-Sakata (PMNS) [10] neutrino mixing matrix, and thus can be directly related to the low energy CP-violation in the lepton sector (e.g. in neutrino oscillations, etc.). The analysis performed in [9] (see also [11, 12]) was stimulated by the progress made in the understanding of the importance of lepton flavour effects in leptogenesis [13, 14, 15, 16, 17, 18]. It led to the realisation that these effects can play crucial role in the leptogenesis scenario of baryon asymmetry generation [15, 16, 17]. It was noticed in [16], in particular, that “Scenarios in which $\epsilon_1 = 0$ while $\epsilon_1^j \neq 0$ entail the possibility that the phases in the light neutrino mixing matrix U are the only source of CP violation.”, ϵ_1^j and ϵ_1 being respectively the individual lepton number and the total lepton number CP violating asymmetries.

As is well-known, the leptogenesis theory [1] is based on the see-saw mechanism of neutrino mass generation [19]. The latter provides a natural explanation of the observed smallness of neutrino masses (see, e.g. [20, 21, 22]). An additional appealing feature of the see-saw scenario is that through the leptogenesis theory it allows to relate the generation and the smallness of neutrino masses with the generation of the baryon (matter-antimatter) asymmetry of the Universe, Y_B .

The non-supersymmetric version of the type I see-saw model with two or three heavy right-handed (RH) Majorana neutrinos is the minimal scheme in which leptogenesis can be implemented. In [9] the analysis was performed within the simplest type I see-saw mechanism of neutrino mass generation with three heavy RH Majorana neutrinos, N_j , $j = 1, 2, 3$. Taking into account the lepton flavour effects in leptogenesis it was shown [9], in particular, that if the heavy Majorana neutrinos have a hierarchical spectrum, i.e. if $M_1 \ll M_{2,3}$, M_j being the mass of N_j , the observed baryon asymmetry Y_B can be produced even if the only source of CP-violation is the Majorana and/or Dirac phase(s) in the PMNS matrix¹ $U_{\text{PMNS}} \equiv U$. Let us recall that in the case of hierarchical spectrum of the heavy Majorana neutrinos, the lepton flavour effects can be significant in leptogenesis provided the mass of the lightest one M_1 satisfies the constraint [15, 16, 17] (see also [23]): $M_1 \lesssim 10^{12}$ GeV. In this case the predicted value of the baryon asymmetry depends explicitly (i.e. directly) on U and on the CP-violating phases in U . The results quoted above were demonstrated to hold both for normal hierarchical (NH) and inverted hierarchical (IH) spectrum of masses of the light Majorana neutrinos (see, e.g. [20]). In both these cases they were obtained for negligible lightest neutrino mass and CP-conserving elements of the orthogonal matrix R , present in the “orthogonal” parametrisation [24] of the matrix

¹The same result was shown to hold also for quasi-degenerate in mass heavy RH Majorana neutrinos [9].

of neutrino Yukawa couplings. The CP-invariance constraints imply [9] that the matrix R could conserve the CP-symmetry if its elements are real or purely imaginary. As was demonstrated in [9], for NH spectrum and negligible lightest neutrino mass m_1 one can have successful thermal leptogenesis with real R . In contrast, in the case of IH spectrum and negligible lightest neutrino mass (m_3), the requisite baryon asymmetry was found to be produced for CP-conserving matrix R only if certain elements of R are purely imaginary: for real R the baryon asymmetry Y_B is strongly suppressed [8] and leptogenesis cannot be successful for $M_1 \lesssim 10^{12}$ GeV (i.e. in the regime in which the lepton flavour effects are significant). It was suggested in [9] that the observed value of Y_B can be reproduced for $M_1 \lesssim 10^{12}$ GeV in the case of IH spectrum and real (CP-conserving) elements of R if the lightest neutrino mass m_3 is non-negligible, having a value in the interval $10^{-2}\sqrt{\Delta m_\odot^2} \lesssim m_3 \lesssim 0.5\sqrt{\Delta m_\odot^2}$, where $\Delta m_\odot^2 = \Delta m_{21}^2 \equiv m_2^2 - m_1^2 \cong 8.0 \times 10^{-5}$ eV² is the mass squared difference responsible for the solar neutrino oscillations, and $m_{1,2}$ are the masses of the two additional light Majorana neutrinos. In this case we still would have $m_3 \ll m_{1,2}$ since $m_{1,2} \cong \sqrt{|\Delta m_A^2|} \cong 5.0 \times 10^{-2}$ eV, $|\Delta m_A^2| \equiv m_2^2 - m_3^2 \cong m_1^2 - m_3^2$ being the mass squared difference associated with the dominant atmospheric neutrino oscillations.

It should be noted that constructing a viable see-saw model which leads to real or purely imaginary matrix R might encounter serious difficulties, as two recent attempts in this direction indicate [18, 25]. However, constructing such a model lies outside the scope of our study.

In the present article we investigate the effects of the lightest neutrino mass on “flavoured” (thermal) leptogenesis. We concentrate on the case when the CP-violation necessary for the generation of the observed baryon asymmetry of the Universe is due exclusively to the Dirac and/or Majorana CP-violating phases in the PMNS matrix U . Our study is performed within the simplest type I see-saw scenario with three heavy RH Majorana neutrinos N_j , $j = 1, 2, 3$. The latter are assumed to have a hierarchical mass spectrum, $M_1 \ll M_{2,3}$. Throughout the present study we employ the “orthogonal” parametrisation of the matrix of neutrino Yukawa couplings [24]. As was already mentioned earlier, this parametrisation involves an orthogonal matrix R , $R^T R = R R^T = \mathbf{1}$. Although, in general, the matrix R can be complex, i.e. CP-violating, in the present work we are primarily interested in the possibility that R conserves the CP-symmetry. We consider the two types of light neutrino mass spectrum allowed by the data (see, e.g. [20]): i) with normal ordering ($\Delta m_A^2 > 0$), $m_1 < m_2 < m_3$, and ii) with inverted ordering ($\Delta m_A^2 < 0$), $m_3 < m_1 < m_2$. The case of inverted hierarchical (IH) spectrum and real (and CP-conserving) matrix R is investigated in detail. Results for the normal hierarchical (NH) spectrum are also presented.

Our analysis is performed for negligible renormalisation group (RG) running of m_j and of the parameters in the PMNS matrix U from M_Z to M_1 . This possibility is realised (in the class of theories of interest) for sufficiently small values of the lightest neutrino mass $\min(m_j)$ [26, 27], e.g., for $\min(m_j) \lesssim 0.10$ eV. The latter condition is fulfilled for the NH and IH neutrino mass spectra, as well as for spectrum with partial hierarchy (see,

e.g. [28]). Under the indicated condition m_j , and correspondingly Δm_A^2 and Δm_\odot^2 , and U can be taken at the scale $\sim M_Z$, at which the neutrino mixing parameters are measured.

Throughout the present work we use the standard parametrisation of the PMNS matrix:

$$U = \begin{pmatrix} c_{12}c_{13} & s_{12}c_{13} & s_{13}e^{-i\delta} \\ -s_{12}c_{23} - c_{12}s_{23}s_{13}e^{i\delta} & c_{12}c_{23} - s_{12}s_{23}s_{13}e^{i\delta} & s_{23}c_{13} \\ s_{12}s_{23} - c_{12}c_{23}s_{13}e^{i\delta} & -c_{12}s_{23} - s_{12}c_{23}s_{13}e^{i\delta} & c_{23}c_{13} \end{pmatrix} \text{diag}(1, e^{i\frac{\alpha_{21}}{2}}, e^{i\frac{\alpha_{31}}{2}}) \quad (1)$$

where $c_{ij} \equiv \cos \theta_{ij}$, $s_{ij} \equiv \sin \theta_{ij}$, $\theta_{ij} = [0, \pi/2]$, $\delta = [0, 2\pi]$ is the Dirac CP-violating (CPV) phase and α_{21} and α_{31} are the two Majorana CPV phases [29, 30], $\alpha_{21,31} = [0, 2\pi]$. All our numerical results are obtained for the current best fit values of the solar and atmospheric neutrino oscillation parameters [31, 32, 33], Δm_\odot^2 , $\sin^2 \theta_{12}$ and Δm_A^2 , $\sin^2 2\theta_{23}$:

$$\Delta m_\odot^2 = \Delta m_{21}^2 = 8.0 \times 10^{-5} \text{ eV}^2, \quad \sin^2 \theta_{12} = 0.30, \quad (2)$$

$$|\Delta m_A^2| = |\Delta m_{31(32)}^2| = 2.5 \times 10^{-3} \text{ eV}^2, \quad \sin^2 2\theta_{23} = 1. \quad (3)$$

In certain cases the predictions for $|Y_B|$ are very sensitive to the variation of $\sin^2 \theta_{12}$ and $\sin^2 2\theta_{23}$ within their 95% C.L. allowed ranges:

$$0.26 \leq \sin^2 \theta_{12} \leq 0.36, \quad 0.36 \leq \sin^2 \theta_{23} \leq 0.64, \quad 95\% \text{ C.L.} \quad (4)$$

We also use the current upper limit on the CHOOZ mixing angle θ_{13} [34, 31, 32]:

$$\sin^2 \theta_{13} < 0.025 \text{ (0.041)}, \quad 95\% \text{ (99.73\%)} \text{ C.L.} \quad (5)$$

2 Baryon Asymmetry from Low Energy CP-Violating Dirac and Majorana Phases in U_{PMNS}

Following [9] we perform the analysis in the framework of the simplest type I see-saw scenario. It includes the Lagrangian of the Standard Model (SM) with the addition of three heavy right-handed Majorana neutrinos N_j ($j = 1, 2, 3$) with masses $0 < M_1 < M_2 < M_3$ and Yukawa couplings λ_{jl} , $l = e, \mu, \tau$. We will work in the basis in which i) the Yukawa couplings for the charged leptons are flavour-diagonal, and ii) the Majorana mass term of the RH neutrino fields is also diagonal. The heavy Majorana neutrinos are assumed to possess a hierarchical mass spectrum, $M_1 \ll M_2 \ll M_3$.

In what follows we will use the well-known ‘‘orthogonal parametrisation’’ of the matrix of neutrino Yukawa couplings [24]:

$$\lambda = \frac{1}{v} \sqrt{M} R \sqrt{m} U^\dagger, \quad (6)$$

where R is, in general, a complex orthogonal matrix, $R R^T = R^T R = \mathbf{1}$, M and m are diagonal matrices formed by the masses of N_j and of the light Majorana neutrinos ν_j ,

$M \equiv \text{Diag}(M_1, M_2, M_3)$, $m \equiv \text{Diag}(m_1, m_2, m_3)$, $M_j > 0$, $m_k \geq 0$, and $v = 174$ GeV is the vacuum expectation value of the Higgs doublet field. We shall assume that the matrix R has real and/or purely imaginary elements.

In the case of “hierarchical” heavy Majorana neutrinos N_j , the CP-violating asymmetries, relevant for leptogenesis, are generated in out-of-equilibrium decays of the lightest one, N_1 . The asymmetry in the lepton flavour l (lepton charge L_l) is given by [15, 16, 17]:

$$\epsilon_l = -\frac{3M_1}{16\pi v^2} \frac{\text{Im} \left(\sum_{jk} m_j^{1/2} m_k^{3/2} U_{lj}^* U_{lk} R_{1j} R_{1k} \right)}{\sum_i m_i |R_{1i}|^2}. \quad (7)$$

Thus, for real or purely imaginary elements R_{1j} of R , $\epsilon_e + \epsilon_\mu + \epsilon_\tau = 0$.

There are three possible regimes of generation of the baryon asymmetry in the leptogenesis scenario [15, 16, 17]. At temperatures $T \sim M_1 > 10^{12}$ GeV the lepton flavours are indistinguishable and the one flavour approximation is valid. The relevant asymmetry is $\epsilon = \epsilon_e + \epsilon_\mu + \epsilon_\tau$ and in the case of interest (real or purely imaginary CP-conserving R_{1j}) no baryon asymmetry is produced. For 10^9 GeV $\lesssim T \sim M_1 \lesssim 10^{12}$ GeV, the Boltzmann evolution of the asymmetry ϵ_τ in the τ -flavour (lepton charge L_τ of the Universe) is distinguishable from the evolution of the $(e + \mu)$ -flavour (or lepton charge $L_e + L_\mu$) asymmetry $\epsilon_e + \epsilon_\mu$. This corresponds to the so-called “two-flavour regime”². At smaller temperatures, $T \sim M_1 \lesssim 10^9$ GeV, the evolution of the μ -flavour (lepton charge L_μ) and of ϵ_μ also become distinguishable (three-flavour regime). The produced baryon asymmetry is a sum of the relevant flavour asymmetries, each weighted by the corresponding efficiency factor accounting for the wash-out processes.

In the two-flavour regime, 10^9 GeV $\lesssim T \sim M_1 \lesssim 10^{12}$ GeV, the baryon asymmetry³ predicted in the case of interest is given by [17] (see also [9]):

$$\begin{aligned} Y_B &\cong -\frac{12}{37g_*} \left(\epsilon_2 \eta \left(\frac{417}{589} \tilde{m}_2 \right) + \epsilon_\tau \eta \left(\frac{390}{589} \tilde{m}_\tau \right) \right) \\ &= -\frac{12}{37g_*} \epsilon_\tau \left(\eta \left(\frac{390}{589} \tilde{m}_\tau \right) - \eta \left(\frac{417}{589} \tilde{m}_2 \right) \right), \end{aligned} \quad (8)$$

where the second expression corresponds to real and purely imaginary $R_{1j}R_{1k}$. Here $g_* = 217/2$ is the number of relativistic degrees of freedom, $\epsilon_2 = \epsilon_e + \epsilon_\mu$, $\tilde{m}_2 = \tilde{m}_e + \tilde{m}_\mu$, \tilde{m}_l is the “wash-out mass parameter” for the asymmetry in the lepton flavour l [15, 16, 17],

$$\tilde{m}_l = \left| \sum_k R_{1k} m_k^{1/2} U_{lk}^* \right|^2, \quad l = e, \mu, \tau, \quad (9)$$

and $\eta(390\tilde{m}_\tau/589) \cong \eta(0.66\tilde{m}_\tau)$ and $\eta(417\tilde{m}_2/589) \cong \eta(0.71\tilde{m}_2)$ are the efficiency factors for generation of the asymmetries ϵ_τ and ϵ_2 . The efficiency factors are well approximated

²As was suggested in [9] and confirmed in the more detailed study [23], in the two-flavour regime of leptogenesis the flavour effects are fully developed at $M_1 \lesssim 5 \times 10^{11}$ GeV.

³The expression we give is of the baryon asymmetry normalised to the entropy density, see, e.g. [9].

by the expression [17]:

$$\eta(X) \cong \left(\frac{8.25 \times 10^{-3} \text{ eV}}{X} + \left(\frac{X}{2 \times 10^{-4} \text{ eV}} \right)^{1.16} \right)^{-1}. \quad (10)$$

At $T \sim M_1 \lesssim 10^9 \text{ GeV}$, the three-flavour regime is realised and [17]

$$Y_B \cong -\frac{12}{37g_*} \left(\epsilon_e \eta\left(\frac{151}{179} \tilde{m}_e\right) + \epsilon_\mu \eta\left(\frac{344}{537} \tilde{m}_\mu\right) + \epsilon_\tau \eta\left(\frac{344}{537} \tilde{m}_\tau\right) \right). \quad (11)$$

For real or purely imaginary $R_{1j}R_{1k}$ of interest, $j \neq k$, it proves convenient to cast the asymmetries ϵ_l in the form [9]:

$$\epsilon_l = -\frac{3M_1}{16\pi v^2} \frac{\sum_k \sum_{j>k} \sqrt{m_k m_j} (m_j - m_k) \rho_{kj} |R_{1k} R_{1j}| \text{Im}(U_{lk}^* U_{lj})}{\sum_i m_i |R_{1i}|^2}, \text{ if } \text{Im}(R_{1k} R_{1j}) = 0, \quad (12)$$

$$\epsilon_l = -\frac{3M_1}{16\pi v^2} \frac{\sum_k \sum_{j>k} \sqrt{m_k m_j} (m_j + m_k) \rho_{kj} |R_{1k} R_{1j}| \text{Re}(U_{lk}^* U_{lj})}{\sum_i m_i |R_{1i}|^2}, \text{ if } \text{Re}(R_{1k} R_{1j}) = 0. \quad (13)$$

where we have used $R_{1j}R_{1k} = \rho_{jk} |R_{1j}R_{1k}|$ and $R_{1j}R_{1k} = i\rho_{jk} |R_{1j}R_{1k}|$, $\rho_{jk} = \pm 1$, $j \neq k$. Note that real (purely imaginary) $R_{1k}R_{1j}$ and purely imaginary (real) $U_{lk}^* U_{lj}$, $j \neq k$, implies violation of CP-invariance by the matrix R [9]. In order for the CP-symmetry to be broken at low energies, we should have both $\text{Re}(U_{lk}^* U_{lj}) \neq 0$ and $\text{Im}(U_{lk}^* U_{lj}) \neq 0$ (see [9] for further details). Note also that if R_{1j} , $j = 1, 2, 3$, is real or purely imaginary, as the condition of CP-invariance requires [9], of the three quantities $R_{11}R_{12}$, $R_{11}R_{13}$ and $R_{12}R_{13}$, relevant for our discussion, not more than two can be purely imaginary, i.e. if, for instance, $R_{11}R_{12} = i\rho_{12} |R_{11}R_{12}|$ and $R_{12}R_{13} = i\rho_{23} |R_{12}R_{13}|$, then we will have $R_{11}R_{13} = \rho_{13} |R_{11}R_{13}|$.

3 Effects of Lightest Neutrino Mass: Real R_{1j}

We consider next the possible effects the lightest neutrino mass $\min(m_j)$ can have on (thermal) leptogenesis. We will assume that the latter takes place in the regime in which the lepton flavour effects are significant and that the CP-violation necessary for the generation of the baryon asymmetry is provided only by the Majorana or Dirac phases in the PMNS matrix U_{PMNS} . In the present Section we analyse the possibility of real elements R_{1j} , $j = 1, 2, 3$, of the matrix R . The study will be performed both for light neutrino mass spectrum with normal and inverted ordering. We begin with the more interesting possibility of spectrum with inverted ordering (hierarchy).

3.1 Light Neutrino Mass Spectrum with Inverted Ordering

The case of inverted hierarchical (IH) neutrino mass spectrum, $m_3 \ll m_1 < m_2$, $m_{1,2} \cong \sqrt{|\Delta m_{\text{A}}^2|}$, is of particular interest since, as was already mentioned in the Introduction,

for real R_{1j} , $j = 1, 2, 3$, IH spectrum and negligible lightest neutrino mass $m_3 \cong 0$, it is impossible to generate the observed baryon asymmetry $Y_B \cong 8.6 \times 10^{-11}$ in the regime of “flavoured” leptogenesis [9], i.e. for $M_1 \lesssim 10^{12}$ GeV, if the only source of CP violation are the Majorana and/or Dirac phases in the PMNS matrix. For $m_3 \ll m_1 < m_2$ and real R_{1j} , the terms proportional to $\sqrt{m_3}$ in the expressions for the asymmetries ϵ_l and wash-out mass parameters \tilde{m}_l , $l = e, \mu, \tau$, will be negligible if $m_3 \cong 0$, or if $R_{13} = 0$ and $R_{11}, R_{12} \neq 0$, $R_{11}^2 + R_{12}^2 = 1$. The main reason for the indicated negative result lies in the fact that if $m_3 = 0$, or $m_3 \ll m_1 < m_2$ and $R_{13} = 0$, the lepton asymmetries ϵ_l are suppressed by the factor $\Delta m_\odot^2 / (2\Delta m_A^2) \cong 1.6 \times 10^{-2}$, while $|R_{11}|, |R_{12}| \leq 1$, and the resulting baryon asymmetry is too small⁴.

In what follows we will analyse the generation of the baryon asymmetry Y_B for real R_{1j} , $j = 1, 2, 3$, when m_3 is non-negligible. We will assume that Y_B is produced in the two-flavour regime, 10^9 GeV $\lesssim M_1 \lesssim 10^{12}$ GeV. Under these conditions the terms $\propto \sqrt{m_3}$ in ϵ_l will be dominant provided [9]

$$2 \left(\frac{m_3}{\sqrt{\Delta m_\odot^2}} \right)^{\frac{1}{2}} \left(\frac{\Delta m_A^2}{\Delta m_\odot^2} \right)^{\frac{3}{4}} \frac{|R_{13}|}{|R_{11(12)}|} \gg 1. \quad (14)$$

This inequality can be fulfilled if $R_{11} \rightarrow 0$, or $R_{12} \rightarrow 0$, and if m_3 is sufficiently large. The neutrino mass spectrum will be of the IH type if m_3 still obeys $m_3 \ll m_{1,2}$. The latter condition can be satisfied for m_3 having a value $m_3 \lesssim 5 \times 10^{-3}$ eV $\ll \sqrt{|\Delta m_A^2|}$. Our general analysis will be performed for values of m_3 from the interval 10^{-10} eV $\lesssim m_3 \lesssim 5 \times 10^{-2}$ eV.

Consider the simple possibility of $R_{11} = 0$. We will present later results of a general analysis, performed without setting R_{11} to 0. For $R_{11} = 0$ the asymmetry $\epsilon_\tau = -(\epsilon_e + \epsilon_\mu)$ of interest is given by:

$$\epsilon_\tau \cong -\frac{3M_1}{16\pi v^2} \sqrt{m_3 m_2} \left(1 - \frac{m_3}{m_2} \right) \rho_{23} r \operatorname{Im} (U_{\tau 2}^* U_{\tau 3}), \quad (15)$$

where

$$m_2 = \sqrt{m_3^2 + |\Delta m_A^2|}, \quad (16)$$

$$r = \frac{|R_{13} R_{12}|}{|R_{12}|^2 + \frac{m_3}{m_2} |R_{13}|^2}, \quad (17)$$

and

$$\operatorname{Im} (U_{\tau 2}^* U_{\tau 3}) = -c_{23} c_{13} \operatorname{Im} (e^{i(\alpha_{31} - \alpha_{21})/2} (c_{12} s_{23} + s_{12} c_{23} s_{13} e^{-i\delta})). \quad (18)$$

The two relevant wash-out mass parameters are given by:

$$\tilde{m}_\tau = m_2 R_{12}^2 |U_{\tau 2}|^2 + m_3 R_{13}^2 |U_{\tau 3}|^2 + 2 \sqrt{m_2 m_3} \rho_{23} |R_{12} R_{13}| \operatorname{Re} (U_{\tau 2}^* U_{\tau 3}), \quad (19)$$

$$\tilde{m}_2 \equiv \tilde{m}_e + \tilde{m}_\mu = m_2 R_{12}^2 + m_3 R_{13}^2 - \tilde{m}_\tau, \quad (20)$$

⁴This suppression is present also in the “one-flavour” regime of Y_B generation, i.e. in the sum $\epsilon_e + \epsilon_\mu + \epsilon_\tau$, when $R_{13} = 0$ and the product $R_{11} R_{12}$ has non-trivial real and imaginary parts [8].

where $\rho_{23} = \text{sgn}(R_{12}R_{13})$.

The orthogonality of the matrix R implies that $R_{11}^2 + R_{12}^2 + R_{13}^2 = 1$, which in the case under consideration reduces to $R_{12}^2 + R_{13}^2 = 1$. It is not difficult to show that for R_{12} and R_{13} satisfying this constraint, the maximum of the function r , and therefore of the asymmetry $|\epsilon_\tau|$, takes place for

$$R_{12}^2 = \frac{m_3}{m_3 + m_2}, \quad R_{13}^2 = \frac{m_2}{m_3 + m_2}, \quad R_{12}^2 < R_{13}^2. \quad (21)$$

At the maximum we get

$$\max(|r|) = \frac{1}{2} \left(\frac{m_2}{m_3} \right)^{\frac{1}{2}} \cong \frac{1}{2} \left(\frac{\sqrt{|\Delta m_A^2|}}{m_3} \right)^{\frac{1}{2}}, \quad (22)$$

and

$$|\epsilon_\tau| \cong \frac{3M_1}{32\pi v^2} (m_2 - m_3) |\text{Im}(U_{\tau 2}^* U_{\tau 3})| \cong \frac{3M_1}{32\pi v^2} \sqrt{|\Delta m_A^2|} |\text{Im}(U_{\tau 2}^* U_{\tau 3})|. \quad (23)$$

The second approximate equalities in eqs. (22) and (23) correspond to IH spectrum, i.e. to $m_3 \ll m_2 \cong \sqrt{|\Delta m_A^2|}$. Thus, the maximum of the asymmetry $|\epsilon_\tau|$ thus found i) is not suppressed by the factor $\Delta m_\odot^2/(\Delta m_A^2)$, and ii) practically does not depend on m_3 in the case of IH spectrum. Given the fact that

$$|\epsilon_\tau| \cong 5.0 \times 10^{-8} \frac{m_2 - m_3}{\sqrt{|\Delta m_A^2|}} \left(\frac{\sqrt{|\Delta m_A^2|}}{0.05 \text{ eV}} \right) \left(\frac{M_1}{10^9 \text{ GeV}} \right) |\text{Im}(U_{\tau 2}^* U_{\tau 3})|, \quad (24)$$

$\max(|\text{Im}(U_{\tau 2}^* U_{\tau 3})|) \cong 0.46$, where we have used $\sin^2 2\theta_{23} = 1$, $\sin^2 \theta_{12} = 0.30$ and $\sin^2 \theta_{13} < 0.04$, and that $\max(|\eta(0.66\tilde{m}_\tau) - \eta(0.71\tilde{m}_2)|) \cong 7 \times 10^{-2}$, we find the absolute upper bound on the baryon asymmetry in the case of IH spectrum and real matrix R (real $R_{1j}R_{1k}$):

$$|Y_B| \lesssim 4.8 \times 10^{-12} \left(\frac{\sqrt{|\Delta m_A^2|}}{0.05 \text{ eV}} \right) \left(\frac{M_1}{10^9 \text{ GeV}} \right). \quad (25)$$

This upper bound allows to determine the minimal value of M_1 for which it is possible to reproduce the observed value of $|Y_B|$ lying in the interval $8.0 \times 10^{-11} \lesssim |Y_B| \lesssim 9.2 \times 10^{-11}$ for IH spectrum, real R and $R_{11} = 0$:

$$M_1 \gtrsim 1.7 \times 10^{10} \text{ GeV}. \quad (26)$$

The values of R_{12} , for which $|\epsilon_\tau|$ is maximal, can differ, in general, from those maximising $|Y_B|$ due to the dependence of the wash-out mass parameters and of the corresponding efficiency factors on R_{12} . However, this difference, when it is present, does not exceed 30%, as our calculations show, and is not significant. At the same time the discussion of the wash-out effects for the maximal $|\epsilon_\tau|$ is rather straightforward and allows to understand in a rather simple way the specific features of the generation of $|Y_B|$ in the case under discussion. For these reasons in our discussion of the wash-out mass parameters we

will use R_{12} maximising $|\epsilon_\tau|$. All our major numerical results and most of the figures are obtained for R_{12} maximising $|Y_B|$.

For R_{12} (R_{13}), which maximises the ratio $|r|$ and the asymmetry $|\epsilon_\tau|$, the relevant wash-out mass parameters are given by:

$$\tilde{m}_\tau = \frac{m_2 m_3}{m_3 + m_2} [|U_{\tau 2}|^2 + |U_{\tau 3}|^2 + 2\rho_{23} \text{Re} (U_{\tau 2}^* U_{\tau 3})] , \quad (27)$$

$$\tilde{m}_2 = 2 \frac{m_2 m_3}{m_3 + m_2} - \tilde{m}_\tau . \quad (28)$$

Equations (24), (27) and (28) suggest that in the case of IH spectrum with non-negligible m_3 , $m_3 \ll \sqrt{|\Delta m_A^2|}$, the generated baryon asymmetry $|Y_B|$ can be strongly enhanced in comparison with the asymmetry $|Y_B|$ produced if $m_3 \cong 0$. The enhancement can be by a factor of ~ 100 . Indeed, the maximum of the asymmetry $|\epsilon_\tau|$ (with respect to $|R_{12}|$), eq. (23), does not contain the suppression factor $\Delta m_\odot^2 / (2\Delta m_A^2) \cong 1.6 \times 10^{-2}$ and its magnitude is not controlled by m_3 , but rather by $\sqrt{|\Delta m_A^2|}$. At the same time, the wash-out mass parameters \tilde{m}_τ and \tilde{m}_2 , eqs. (27) and (28), are determined by $m_2 m_3 / (m_2 + m_3) \cong m_3$. The latter in the case under discussion can take values as large as $m_3 \sim 5 \times 10^{-3}$ eV. The efficiency factors $\eta(0.66\tilde{m}_\tau)$ and $\eta(0.71\tilde{m}_2)$, which enter into the expression for the baryon asymmetry, eq. (8), have a maximal value $\eta(X) \cong (6 - 7) \times 10^{-2}$ when $X \cong (0.7 - 1.5) \times 10^{-3}$ eV (weak wash-out regime). Given the range of values of m_3 for IH spectrum extends to $\sim 5 \times 10^{-3}$ eV, one can always find a value of m_3 in this range such that \tilde{m}_τ or \tilde{m}_2 take a value maximising $\eta(0.66\tilde{m}_\tau)$ or $\eta(0.71\tilde{m}_2)$, and $|\eta(0.66\tilde{m}_\tau) - \eta(0.71\tilde{m}_2)|$. This qualitative discussion suggests that there always exists an interval of values of m_3 for which the baryon asymmetry is produced in the weak wash-out regime. On the basis of the above considerations one can expect that we can have successful leptogenesis for a non-negligible m_3 in the case of IH spectrum even if the requisite CP-violation is provided by the Majorana or Dirac phase(s) in the PMNS matrix. This is confirmed by the detailed (analytic and numerical) analysis we have performed, the results of which are described below.

A. Leptogenesis due to Majorana CP-Violation in U_{PMNS}

We will assume first that the Dirac phase δ has a CP-conserving value, $\delta = 0$; π . For $\delta = 0$ (π), we have $|\text{Im}(U_{\tau 2}^* U_{\tau 3})| = c_{23} c_{13} (s_{23} c_{12} \begin{smallmatrix} + \\ - \end{smallmatrix} c_{23} s_{12} s_{13}) |\sin \alpha_{32}/2|$ and correspondingly $0.36 |\sin \alpha_{32}/2| \lesssim |\text{Im}(U_{\tau 2}^* U_{\tau 3})| \lesssim 0.46 |\sin \alpha_{32}/2|$, where $\alpha_{32} = \alpha_{31} - \alpha_{21}$ and we have used the best fit values of $\sin^2 2\theta_{23}$ and $\sin^2 \theta_{12}$, and the limit $\sin^2 \theta_{13} < 0.04$. For $s_{13} = 0$ we get: $|\text{Im}(U_{\tau 2}^* U_{\tau 3})| \cong 0.42 |\sin \alpha_{32}/2|$. The terms proportional to s_{13} have a subdominant effect on the magnitude of the calculated $|\epsilon_\tau|$ and $|Y_B|$.

It is easy to check that the asymmetry $|\epsilon_\tau|$ and the wash-out mass parameters $\tilde{m}_{\tau,2}$ remain invariant with respect to the change $\rho_{23} \rightarrow -\rho_{23}$, $\alpha_{32} \rightarrow 2\pi - \alpha_{32}$. Thus, the baryon asymmetry $|Y_B|$ satisfies the following relation: $|Y_B(\rho_{23}, \alpha_{32})| = |Y_B(-\rho_{23}, 2\pi - \alpha_{32})|$. Therefore, unless otherwise stated, we will consider the case of $\rho_{23} = +1$ in what follows.

The absolute maximum of the asymmetry $|Y_B|$ with respect to α_{32} is not obtained for $\alpha_{32} = \pi$ for which $|\epsilon_\tau|$ is maximal⁵, but rather for α_{32} having a value in the interval $\alpha_{32} \cong (\pi/2 - 2\pi/3)$ if $\rho_{23} = +1$, or in the interval $\alpha_{32} \cong (4\pi/3 - 3\pi/2)$ if $\rho_{23} = -1$. The maximal value of $|Y_B|$ at $\alpha_{32} = \pi$ is smaller at least by a factor of ~ 2 than the value of $|Y_B|$ at its absolute maximum (see further Fig. 3). As can be easily shown, for $\alpha_{32} \sim \pi$ there is a rather strong mutual compensation between the asymmetries in the lepton charges L_τ and $(L_e + L_\mu)$ owing to the fact that, due to $\text{Re}(U_{\tau 2}^* U_{\tau 3}) = 0$, \tilde{m}_τ and \tilde{m}_2 have relatively close values and $|\eta(0.66\tilde{m}_\tau) - \eta(0.71\tilde{m}_2)| \lesssim 10^{-2}$. Actually, in certain cases one can even have $|\eta(0.66\tilde{m}_\tau) - \eta(0.71\tilde{m}_2)| \cong 0$, and thus $|Y_B| \cong 0$, for α_{32} lying in the interval $\alpha_{32} \sim (\pi - 4\pi/3)$ (see further Fig. 3). Similar cancellation can occur for $s_{13} = 0.2$ at $\alpha_{32} \sim \pi/6$. Obviously, we have $|Y_B| = 0$ for $\alpha_{32} = 0; 2\pi$.

We are interested primarily in the dependence of $|Y_B|$ on m_3 . As m_3 increases from the value of 10^{-10} eV up to 10^{-4} eV, in the case of $R_{11} = 0$ under discussion the maximal possible $|Y_B|$ for a given M_1 increases monotonically, starting from a value which for $M_1 \leq 10^{12}$ GeV is much smaller than the observed one, $\max(|Y_B|) \ll 8.6 \times 10^{-11}$. At approximately $m_3 \cong 2 \times 10^{-6}$ eV, we have $\max(|Y_B|) \cong 8.6 \times 10^{-11}$ for $M_1 \cong 5 \times 10^{11}$ GeV. As m_3 increases beyond 2×10^{-6} eV, $\max(|Y_B|)$ for a given M_1 continues to increase until it reaches a maximum. This maximum occurs for m_3 such that $0.71\tilde{m}_2 \cong 9.0 \times 10^{-4}$ eV and $\eta(0.71\tilde{m}_2)$ is maximal, $\eta(0.71\tilde{m}_2) \cong 6.8 \times 10^{-2}$, while $\eta(0.66\tilde{m}_\tau)$ is considerably smaller. As can be shown, for $\rho_{23} = +1$, it always takes place at $\alpha_{32} \cong \pi/2$. For $\alpha_{32} = \pi/2$, $s_{13} = 0$ and $\rho_{23} = +1$, the maximum of $|Y_B|$ in question is located at $m_3 \cong 7 \times 10^{-4}$ eV. It corresponds to the CP-asymmetry being predominantly in the $(e + \mu)$ -flavour. As m_3 increases further, $|\eta(0.66\tilde{m}_\tau) - \eta(0.71\tilde{m}_2)|$ and correspondingly $|Y_B|$, rapidly decrease. At certain value of m_3 , typically lying in the interval $m_3 \sim (1.5 - 2.5) \times 10^{-3}$ eV, one has $|\eta(0.66\tilde{m}_\tau) - \eta(0.71\tilde{m}_2)| \cong 0$ and $|Y_B|$ goes through a deep minimum: one can have even $|Y_B| = 0$. This minimum of $|Y_B|$ corresponds to a partial or complete cancellation between the asymmetries in the τ -flavour and in the $(e + \mu)$ -flavour⁶. In our example of $\alpha_{32} = \pi/2$, $s_{13} = 0$ and $\rho_{23} = +1$, the indicated minimum of $|Y_B|$ occurs at $m_3 \cong 2.3 \times 10^{-3}$ eV. As m_3 increases further, $|\eta(0.66\tilde{m}_\tau) - \eta(0.71\tilde{m}_2)|$ and $|Y_B|$ rapidly increase and $|Y_B|$ reaches a second maximum, which in magnitude is of the order of the first one. This maximum corresponds to the CP-asymmetry being predominantly in the τ -flavour rather than in the $(e + \mu)$ -flavour: now $\eta(0.66\tilde{m}_\tau)$ is maximal having a value $\eta(0.66\tilde{m}_\tau) \cong 6.8 \times 10^{-2}$, and $\eta(0.71\tilde{m}_2)$ is substantially smaller. For $\rho_{23} = +1$, $s_{13} = 0$ or $s_{13} = 0.2$ and $\delta = 0$, it takes place at a value of α_{32} close to $\pi/2$, $\alpha_{32} \cong \pi/2$, while for $s_{13} = 0.2$ and $\delta = \pi$, it occurs at $\alpha_{32} \cong 2\pi/3$. In the case of $\rho_{23} = +1$, $s_{13} = 0$ and $\alpha_{32} = \pi/2$, the second maximum of $|Y_B|$ is located at $m_3 \cong 7 \times 10^{-3}$ eV. As m_3 increases further, $|Y_B|$ decreases rather slowly monotonically.

These features of the dependence of $|Y_B|$ on m_3 are confirmed by a more general analysis in which, in particular, the value of R_{11} was not set to zero a priori. The results

⁵We would like to recall that in the case of $\alpha_{32} = \pi$, $\delta = 0$; π , and real $R_{1j}R_{1k}$, the requisite violation of CP-symmetry in leptogenesis is provided by the matrix R [9].

⁶A general discussion of the possibility of such a suppression of the baryon asymmetry Y_B in the case of “flavoured” leptogenesis is given in [9].

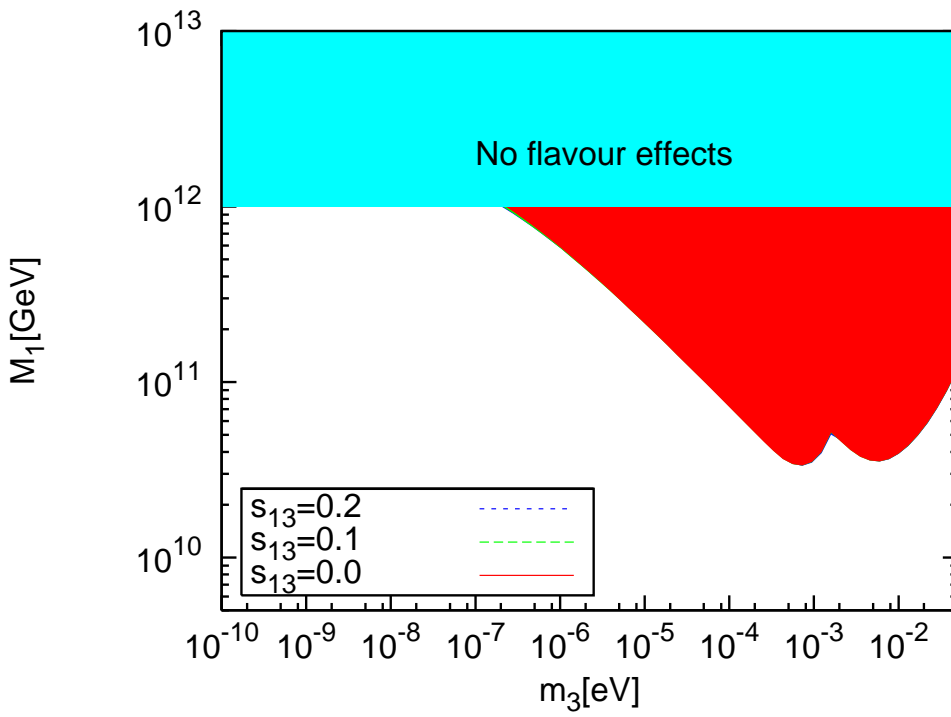


Figure 1: Values of m_3 and M_1 for which the “flavoured” leptogenesis is successful, generating baryon asymmetry $|Y_B| = 8.6 \times 10^{-11}$ (red/dark shaded area). The figure corresponds to hierarchical heavy Majorana neutrinos, light neutrino mass spectrum with inverted ordering (hierarchy), $m_3 < m_1 < m_2$, and real elements R_{1j} of the matrix R . The minimal value of M_1 at given m_3 , for which the measured value of $|Y_B|$ is reproduced, corresponds to CP-violation due to the Majorana phases in the PMNS matrix. The results shown are obtained using the best fit values of neutrino oscillation parameters: $\Delta m_{\odot}^2 = 8.0 \times 10^{-5} \text{ eV}^2$, $\Delta m_{\text{A}}^2 = 2.5 \times 10^{-3} \text{ eV}^2$, $\sin^2 \theta_{12} = 0.30$ and $\sin^2 2\theta_{23} = 1$. See text for further details.

of this analysis are presented in Fig. 1, while Fig. 2 illustrates the dependence of $|Y_B|$ on m_3 in the case of $R_{11} = 0$.

In Fig. 1 we show the correlated values of M_1 and m_3 for which one can have successful leptogenesis in the case of neutrino mass spectrum with inverted ordering and CP-violation due to the Majorana and Dirac phases in U_{PMNS} . The figure was obtained by performing, for given m_3 from the interval $10^{-10} \leq m_3 \leq 0.05 \text{ eV}$, a thorough scan of the relevant parameter space searching for possible enhancement or suppression of the baryon asymmetry with respect to that found for $m_3 = 0$. The real elements of the R -matrix of interest, R_{1j} , $j = 1, 2, 3$, were allowed to vary in their full ranges determined by the condition of orthogonality of the matrix R : $R_{11}^2 + R_{12}^2 + R_{13}^2 = 1$. In particular, R_{11} was *not set to zero*. The Majorana phases $\alpha_{21,31}$ were varied in the interval $[0, 2\pi]$. The calculations were performed for three values of the CHOOZ angle θ_{13} , corresponding to $\sin \theta_{13} = 0; 0.1; 0.2$. In the cases of $\sin \theta_{13} \neq 0$, the Dirac phase δ was allowed to take values in the interval $[0, 2\pi]$. The heavy Majorana neutrino mass M_1 was varied in the interval $10^9 \text{ GeV} \leq M_1 \leq 10^{12} \text{ GeV}$. For given m_3 , the minimal value of the mass

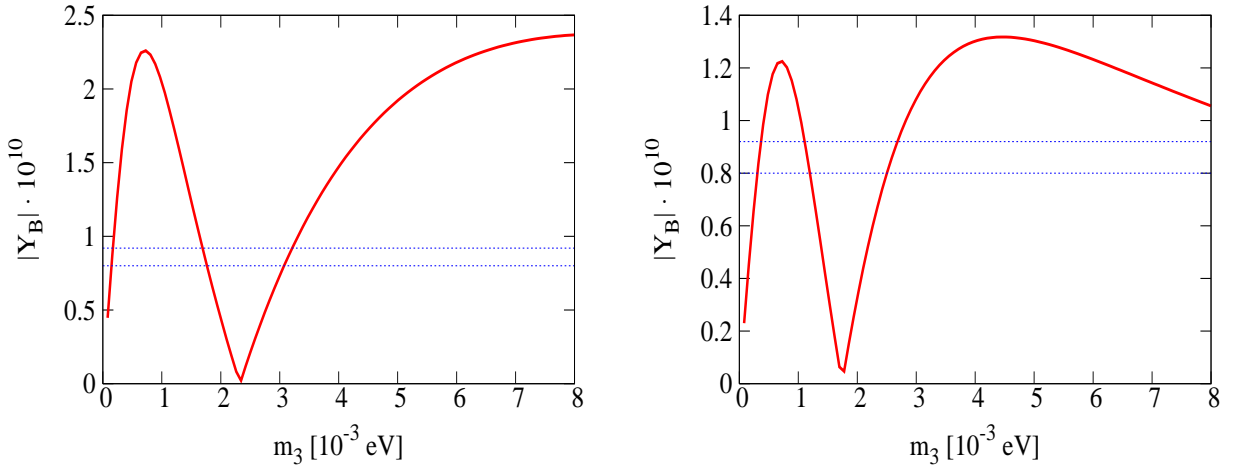


Figure 2: The dependence of $|Y_B|$ on m_3 in the case of IH spectrum, real $R_{1j}R_{1k}$, Majorana CP-violation, $R_{11} = 0$, $\alpha_{32} = \pi/2$, $s_{13} = 0$, $M_1 = 10^{11}$ GeV, and for i) $\text{sgn}(R_{12}R_{13}) = +1$ (left panel), and ii) $\text{sgn}(R_{12}R_{13}) = -1$ (right panel). The baryon asymmetry $|Y_B|$ was calculated for a given m_3 , using the value of $|R_{12}|$, for which the asymmetry $|\epsilon_\tau|$ is maximal. The horizontal dotted lines indicate the allowed range of $|Y_B|$, $|Y_B| = (8.0 - 9.2) \times 10^{-11}$.

M_1 , for which the leptogenesis is successful, generating $|Y_B| \cong 8.6 \times 10^{-11}$, was obtained for the values of the other parameters which maximise $|Y_B|$. The $\min(M_1)$ thus found does not exhibit any significant dependence on s_{13} . If $m_3 \lesssim 2.5 \times 10^{-7}$ eV, leptogenesis cannot be successful for $M_1 \leq 10^{12}$ GeV: the baryon asymmetry produced in this regime is too small. As m_3 increases starting from the indicated value, the maximal $|Y_B|$ for a given $M_1 \leq 10^{12}$ GeV, increases monotonically. Correspondingly, the $\min(M_1)$ for which one can have successful leptogenesis decreases monotonically and for $m_3 \gtrsim 5 \times 10^{-6}$ eV we have $\min(M_1) \lesssim 5 \times 10^{11}$ GeV. The first maximum of $|Y_B|$ (minimum of M_1) as m_3 increases is reached at $m_3 \cong 5.5 \times 10^{-4}$ eV, $\alpha_{32} \cong \pi/2$ ($\alpha_{21} \cong 0.041$, $\alpha_{31} \cong 1.65$), $R_{11} \cong -0.061$, $R_{12} \cong 0.099$, and $R_{13} \cong 0.99$. At the maximum we have $|Y_B| = 8.6 \times 10^{-11}$ for $M_1 \cong 3.4 \times 10^{10}$ GeV. The second maximum of $|Y_B|$ (or minimum of M_1) seen in Fig. 1 corresponds to $m_3 \cong 5.9 \times 10^{-3}$ eV, $\alpha_{32} \cong \pi/2$ ($\alpha_{21} \cong -0.022$, $\alpha_{31} \cong 1.45$), $R_{11} \cong -0.18$, $R_{12} \cong 0.29$, and $R_{13} \cong -0.94$. The observed value of $|Y_B|$ is reproduced in this case for $M_1 \cong 3.5 \times 10^{10}$ GeV.

Similar features are seen in Fig. 2, which shows the dependence of $|Y_B|$ on m_3 for $R_{11} = 0$, fixed $M_1 = 10^{11}$ GeV, $\alpha_{32} = \pi/2$, $s_{13} = 0$ and $\rho_{23} = +1$; (-1) . In the case of $\alpha_{32} = \pi/2$, $s_{13} = 0.2$, $\delta = 0$ and $\rho_{23} = +1$, the absolute maximum of $|Y_B|$ is obtained for $m_3 \cong 6.7 \times 10^{-3}$ eV and $|R_{12}| = 0.34$ (Fig. 3). At this maximum we have $\eta(0.66\tilde{m}_\tau) \cong 0.067$, $\eta(0.71\tilde{m}_2) \cong 0.013$, and

$$|Y_B| \cong 2.6 \times 10^{-12} \left(\frac{\sqrt{|\Delta m_A^2|}}{0.05 \text{ eV}} \right) \left(\frac{M_1}{10^9 \text{ GeV}} \right). \quad (29)$$

Correspondingly, the observed baryon asymmetry $|Y_B|$, $8.0 \times 10^{-11} \lesssim |Y_B| \lesssim 9.2 \times 10^{-11}$, can be reproduced if $M_1 \gtrsim 3.0 \times 10^{10}$ GeV. If $s_{13} = 0$, the same result holds for $M_1 \gtrsim 3.5 \times$

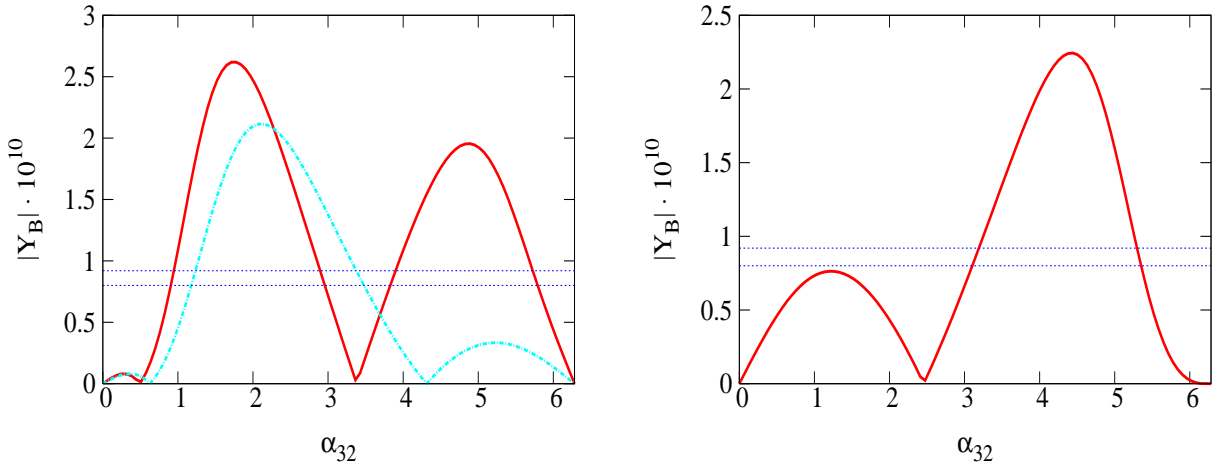


Figure 3: The dependence of $|Y_B|$ on α_{32} (Majorana CP-violation), in the case of IH spectrum, real $R_{1j}R_{1k}$, $R_{11} = 0$, $M_1 = 10^{11}$ GeV, and for i) $s_{13} = 0.2$, $\delta = 0$ (π), $|R_{12}| = 0.34$ (0.38), $m_3 = 6.7$ (4.3) $\times 10^{-3}$ eV, $\text{sgn}(R_{12}R_{13}) = +1$ (left panel, red (blue) line), and ii) $s_{13} = 0$, $\text{sgn}(R_{12}R_{13}) = -1$, $|R_{12}| = 0.41$, $m_3 = 4.2 \times 10^{-3}$ eV (right panel). The values of m_3 and $|R_{12}|$ used maximise $|Y_B|$ at i) $\alpha_{32} = \pi/2$ ($2\pi/3$) and ii) $\alpha_{32} = 3\pi/2$. The horizontal dotted lines indicate the allowed range of $|Y_B| = (8.0 - 9.2) \times 10^{-11}$.

10^{10} GeV. The minimal values of M_1 thus found are somewhat smaller than $\min(M_1) \cong 5.3 \times 10^{10}$ GeV obtained in the case of negligible $m_3 \cong 0$ ($R_{13} = 0$) and purely imaginary $R_{11}R_{12}$ [9]. The dependence of the baryon asymmetry on α_{32} in the case of $s_{13} = 0$; 0.2 discussed above is illustrated in Fig. 3.

One obtains similar results in the case of $R_{12} = 0$, $R_{11}, R_{13} \neq 0$, we have also analysed. The corresponding formulae can be obtained from those derived for $R_{11} = 0$ by replacing R_{12} with R_{11} , $U_{\tau 2}^*$ with $U_{\tau 1}^*$ and m_2 with $m_1 = \sqrt{m_3^2 + |\Delta m_A^2| - \Delta m_\odot^2} \cong \sqrt{m_3^2 + |\Delta m_A^2|} = m_2$. In this case we have, in particular, $|\epsilon_\tau| \propto |\text{Im}(U_{\tau 1}^* U_{\tau 3})| = |c_{23}c_{13}(s_{12}s_{23} \mp c_{12}c_{23}s_{13}) \sin \alpha_{31}/2|$, where the minus (plus) sign corresponds to $\delta = 0$ (π). Evidently, the relevant Majorana phase⁷ is $\alpha_{31}/2$. Numerically we get $0.19|\sin \alpha_{31}/2| \lesssim |\text{Im}(U_{\tau 1}^* U_{\tau 3})| \lesssim 0.35|\sin \alpha_{31}/2|$, while for $s_{13} = 0$ we find $|\text{Im}(U_{\tau 1}^* U_{\tau 3})| \cong 0.27|\sin \alpha_{31}/2|$. Thus, the maximal value of $|\epsilon_\tau|$ for $R_{12} = 0$ is smaller approximately by a factor of 1.3 than the maximal value of $|\epsilon_\tau|$ when $R_{11} = 0$. As a consequence, the minimal M_1 for which we can have successful leptogenesis can be expected to be bigger by a factor of ~ 1.3 than the one we have obtained in the case of $R_{11} = 0$. This is confirmed by the numerical calculations we have performed. For example, for $s_{13} = 0.2$, $\delta = \pi$, $\text{sgn}(R_{12}R_{13}) = -1$, and values of $|R_{11}| = 0.38$ and $m_3 = 4.5 \times 10^{-3}$ eV (which maximise $|Y_B|$ at $\alpha_{31} = 2\pi/3$), we get

$$\max(|Y_B|) \cong 2.2 \times 10^{-12} \left(\frac{\sqrt{|\Delta m_A^2|}}{0.05 \text{ eV}} \right) \left(\frac{M_1}{10^9 \text{ GeV}} \right). \quad (30)$$

⁷Note that the Majorana phase α_{32} ($R_{11} = 0$) or α_{31} ($R_{12} = 0$), relevant for leptogenesis in the case of IH spectrum and real matrix R , does not coincide with the Majorana phase α_{21} , which together with $\sqrt{|\Delta m_A^2|}$ and $\sin^2 \theta_{12}$ determines the values of the effective Majorana mass in neutrinoless double beta decay (see, e.g. [35, 28, 36]).

Therefore the observed value of $|Y_B|$ can be reproduced for $M_1 \gtrsim 3.7 \times 10^{10}$ GeV.

We would like to stress that the results we have obtained in the case of $R_{1j} \neq 0$, $j = 1, 2, 3$, which are shown in Fig. 1, are very different from the results for, e.g. $R_{11} = 0$ and $R_{12}, R_{13} \neq 0$. Along the line of minimal values of M_1 in Fig. 1, for which we can have successful leptogenesis, we find that either $\tilde{m}_2 \sim 10^{-3}$ eV and $\tilde{m}_\tau \sim 2 \times 10^{-4}$ eV, or $\tilde{m}_\tau \sim 2 \times 10^{-3}$ eV and $\tilde{m}_2 \gg 10^{-3}$ eV, practically for any m_3 from the interval⁸ 10^{-10} eV $\leq m_3 \leq 5.0 \times 10^{-2}$ eV. This explains why one can have successful leptogenesis for $\min(M_1) \lesssim 5 \times 10^{11}$ GeV even when $m_3 \cong 5 \times 10^{-6}$ eV. If $R_{11} = 0$, we get for $m_3 \ll m_2$ and $R_{12}(R_{13})$ which maximises the asymmetry $|\epsilon_\tau|$, as it follows from eqs. (27) and (28), $\tilde{m}_\tau \sim \tilde{m}_2 \sim m_3$. Consequently, for $m_3 \ll 10^{-3}$ eV, one also has $\tilde{m}_\tau, \tilde{m}_2 \ll 10^{-3}$ eV, and for $M_1 < 10^{12}$ GeV the baryon asymmetry generated under these conditions is strongly suppressed, $|Y_B| \ll 8.6 \times 10^{-11}$.

B. Dirac CP-Violation in U_{PMNS} and Leptogenesis

We will assume that $\alpha_{21} = \alpha_{31} = 0$ (or $\alpha_{21} = 2\pi k$, $\alpha_{31} = 2\pi k'$, $k, k' = 0, 1, 2, \dots$) and⁹ analyse the simple possibility of $R_{11} = 0$. For $R_{11} = 0$ and $\alpha_{32} = 0$ we have: $|\epsilon_\tau| \propto |\text{Im}(U_{\tau 2}^* U_{\tau 3})| = c_{23}^2 c_{13} s_{12} s_{13} |\sin \delta| \lesssim 0.054 |\sin \delta|$, where we have used $c_{23}^2 = 0.5$ and¹⁰ $s_{12} \cong 0.55$. Thus, for given M_1 the maximal asymmetry $|Y_B|$ we can obtain will be smaller by a factor $\sim (7 - 8)$ than the maximal possible asymmetry $|Y_B|$ in the corresponding case of CP-violation due to the Majorana phase(s) in U_{PMNS} . The wash-out mass parameter \tilde{m}_τ , corresponding to R_{12} maximising $|\epsilon_\tau|$, is given by

$$\tilde{m}_\tau \cong \frac{m_2 m_3}{m_3 + m_2} [(c_{12} s_{23} - \rho_{23} c_{13} c_{23})^2 + s_{12}^2 s_{13}^2 c_{23}^2 + 2 s_{12} s_{13} c_{23} (c_{12} s_{23} - \rho_{23} c_{13} c_{23}) \cos \delta], \quad (31)$$

while \tilde{m}_2 is determined by eq. (28). Depending on the value of ρ_{23} , there are two quite different cases to be considered.

If $\rho_{23} = -1$, the terms $s_{12}^2 s_{13}^2 c_{23}^2$ and $\propto 2 s_{12} s_{13} c_{23} \cos \delta$ in the expression for \tilde{m}_τ , eq. (31), are subdominant and can be neglected¹¹. Thus, \tilde{m}_τ and \tilde{m}_2 practically do not depend on δ and we have for $c_{23} = s_{23} = 1/\sqrt{2}$: $\tilde{m}_\tau \cong 0.5(c_{12} + c_{13})^2 m_2 m_3 / (m_2 + m_3) \cong 1.66 m_2 m_3 / (m_2 + m_3)$, $\tilde{m}_2 \cong 0.34 m_2 m_3 / (m_3 + m_2)$. Both the asymmetry $|\epsilon_\tau|$ and $|Y_B|$ are maximal for $\delta = \pi/2 + k\pi$, $k = 0, 1, \dots$. The dependence of $|Y_B|$ on m_3 is analogous

⁸As our numerical calculations show, at $m_3 \gtrsim 10^{-9}$ eV, for instance, there is a very narrow interval of values of m_3 around $m_3 \sim 2 \times 10^{-3}$ eV, in which both \tilde{m}_2 and \tilde{m}_τ increase rapidly monotonically from $\sim 10^{-3}$ eV and $\sim 2 \times 10^{-4}$ eV to $\sim 7 \times 10^{-3}$ eV and $\sim 2 \times 10^{-3}$ eV, respectively. As m_3 increases from $\sim 2.5 \times 10^{-3}$ eV to $\sim 5.0 \times 10^{-2}$ eV, \tilde{m}_2 continues to increase monotonically, while \tilde{m}_τ remains practically constant.

⁹If in the case of real $R_{1j} R_{1k}$ and, e.g. $R_{11} = 0$ ($R_{12} = 0$), the Majorana phase $\alpha_{32(31)}$ entering into the expression for ϵ_τ takes the CP-conserving value $\alpha_{32(31)} = \pi$, the CP-symmetry will be violated not only by the Dirac phase $\delta \neq k\pi$, $k = 0, 1, 2, \dots$, but also by the matrix R [9].

¹⁰Given the 2σ allowed range of values of $c_{23}^2 = (0.36 - 0.64)$, it is clear that the asymmetry $|\epsilon_\tau|$ can be bigger (smaller) by a factor ~ 1.3 (0.72).

¹¹The term $\propto 2 s_{12} s_{13} c_{23} \cos \delta$, for instance, gives a relative contribution to \tilde{m}_τ not exceeding 10%.

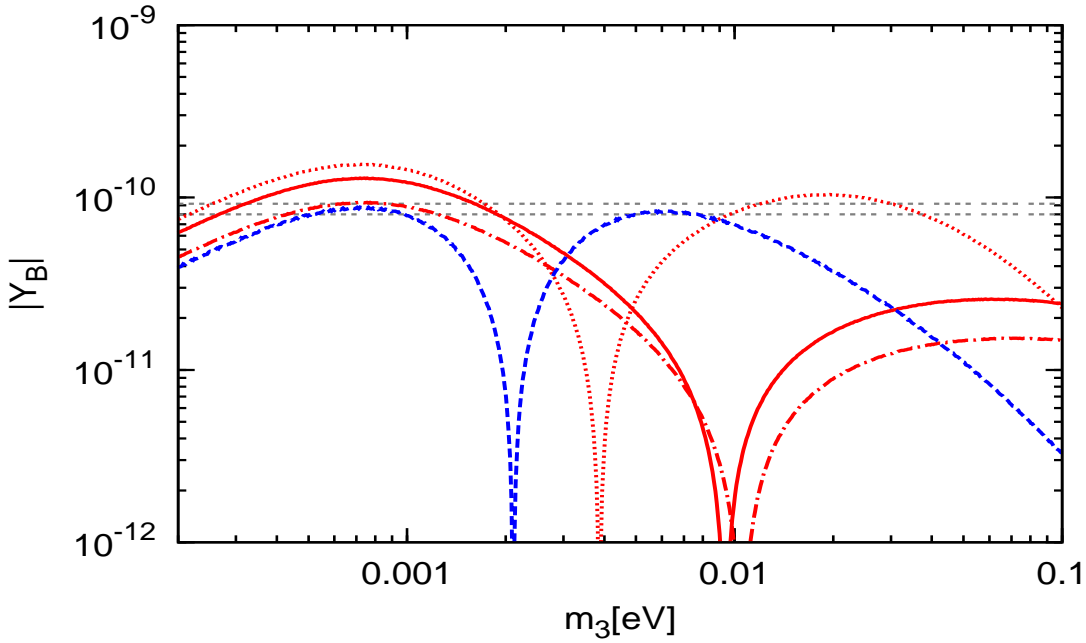


Figure 4: The dependence of $|Y_B|$ on m_3 in the case of spectrum with inverted ordering (hierarchy), real $R_{1j}R_{1k}$ and Dirac CP-violation, for $R_{11} = 0$, $\delta = \pi/2$, $s_{13} = 0.2$, $\alpha_{32} = 0$, $M_1 = 2.5 \times 10^{11}$ GeV and $\text{sgn}(R_{12}R_{13}) = +1$ (-1) (red lines (blue dashed line)). The baryon asymmetry $|Y_B|$ was calculated for a given m_3 , using the value of $|R_{12}|$, for which the CP-asymmetry $|\epsilon_\tau|$ is maximal. The results shown for $\text{sgn}(R_{12}R_{13}) = +1$ are obtained for $\sin^2 \theta_{23} = 0.50$; 0.36 ; 0.64 (red solid, dotted and dash-dotted lines), while those for $\text{sgn}(R_{12}R_{13}) = -1$ correspond to $\sin^2 \theta_{23} = 0.5$. See text for further details.

to that in the case of CP-violation due to the Majorana phase(s) in U_{PMNS} : we have two similar maxima corresponding to the CP-asymmetry being predominantly respectively in the τ -flavour and in the $(e + \mu)$ -flavour. The two maxima are separated by a deep minimum of $|Y_B|$ (Fig. 4). The maxima occur at $m_3 \cong 7.5 \times 10^{-4}$ eV ($|R_{12}| \cong 0.12$) and at $m_3 \cong 4.9 \times 10^{-3}$ eV ($|R_{12}| \cong 0.30$), i.e. at values of m_3 which differ by a factor¹² of ~ 7 . At the first (second) maximum we have $\eta(0.66\tilde{m}_\tau) - \eta(0.71\tilde{m}_2) \cong 0.044$ (-0.046) and

$$|Y_B| \cong 3.5 \text{ (3.7)} \times 10^{-13} |\sin \delta| \left(\frac{\sin \theta_{13}}{0.2} \right) \left(\frac{\sqrt{|\Delta m_A^2|}}{0.05 \text{ eV}} \right) \left(\frac{M_1}{10^9 \text{ GeV}} \right). \quad (32)$$

Thus, the measured value of $|Y_B|$, $8.0 \times 10^{-11} \lesssim |Y_B| \lesssim 9.2 \times 10^{-11}$, can be reproduced for $M_1 \gtrsim 2.3$ (2.2) $\times 10^{11}$ GeV. The flavour effects in leptogenesis are fully developed for $M_1 \lesssim 5 \times 10^{11}$ GeV. This upper bound and the requirement of successful leptogenesis in the case of breaking of CP-symmetry due to the Dirac phase in U_{PMNS} , lead to the

¹²The positions of the two maxima and of the deep minimum of $|Y_B|$ under discussion exhibit very weak dependence on $\sin^2 \theta_{23}$ when the latter is varied within its 2σ allowed range, while the value of $|Y_B|$ at each of the two maxima changes according to $|Y_B| \propto \cos^2 \theta_{23}$.

following lower limit on $|\sin \theta_{13} \sin \delta|$ and thus on $\sin \theta_{13}$:

$$|\sin \theta_{13} \sin \delta| \gtrsim 0.087, \quad \sin \theta_{13} \gtrsim 0.087. \quad (33)$$

The preceding lower bound corresponds to

$$|J_{\text{CP}}| \gtrsim 0.02, \quad (34)$$

where J_{CP} is the rephasing invariant associated with the Dirac phase δ , which controls the magnitude of CP-violation effects in neutrino oscillations¹³ [38]. Values of s_{13} in the range given in eq. (33) can be probed in the forthcoming Double CHOOZ [39] and future reactor neutrino experiments¹⁴ [40]. CP-violation effects with magnitude determined by $|J_{\text{CP}}|$ satisfying (34) are within the sensitivity of the next generation of neutrino oscillation experiments, designed to search for CP- or T- symmetry violations in the oscillations [41]. Since in the case under discussion the wash-out factor $|\eta_B| \equiv |\eta(0.66\tilde{m}_\tau) - \eta(0.71\tilde{m}_2)|$ in the expression for $|Y_B|$ practically does not depend on s_{13} and δ , while both $|Y_B| \propto |s_{13} \sin \delta|$ and $|J_{\text{CP}}| \propto |s_{13} \sin \delta|$, there is a direct relation between $|Y_B|$ and $|J_{\text{CP}}|$ for given m_3 (or m_2) and M_1 :

$$|Y_B| \cong 1.8 \times 10^{-10} |J_{\text{CP}}| |\eta_B| \frac{m_2 - m_3}{\sqrt{|\Delta m_A^2|}} \left(\frac{\sqrt{|\Delta m_A^2|}}{0.05 \text{ eV}} \right) \left(\frac{M_1}{10^9 \text{ GeV}} \right), \quad (35)$$

where $\eta_B = \eta_B(m_2 m_3 / (m_2 + m_3), \theta_{12}, \theta_{23})$ and we have used the best fit values of $\sin^2 \theta_{12}$ and $\sin^2 \theta_{23}$. In the case of IH spectrum we have $(m_2 - m_3) / \sqrt{|\Delta m_A^2|} \cong 1$ and $m_2 m_3 / (m_2 + m_3) \cong m_3$. Similar relation between $|Y_B|$ and $|J_{\text{CP}}|$ holds in an analogous case of normal hierarchical light neutrino mass spectrum [9].

We get somewhat different results if $\rho_{23} = +1$. Now there is a strong compensation between the terms in the round brackets in the expression (31) for \tilde{m}_τ and we have $\tilde{m}_\tau \ll m_2 m_3 / (m_2 + m_3)$. Correspondingly, one has $\tilde{m}_2 \cong 2m_2 m_3 / (m_2 + m_3) \gg \tilde{m}_\tau$. Thus, \tilde{m}_2 practically does not depend on δ and on the neutrino mixing angles. The two wash-out mass parameters \tilde{m}_2 and \tilde{m}_τ can differ by a factor ~ 100 . Indeed, for $s_{23}^2 = c_{23}^2 = 0.5$ and $s_{13} = 0.2$ and $s_{12}^2 = 0.30$ one finds $\tilde{m}_\tau / \tilde{m}_2 \cong 0.5(0.0162 - 0.0156 \cos \delta)$. For fixed $\sin^2 \theta_{12} = 0.30$, the magnitude of the ratio $\tilde{m}_\tau / \tilde{m}_2$ (which is practically independent of $m_2 m_3 / (m_2 + m_3)$) is very sensitive to the value of θ_{23} : for $s_{23}^2 = 0.64$ we get $\tilde{m}_\tau / \tilde{m}_2 \cong 0.5(0.0066 + 0.0043 s_{13}^2 / 0.04 - 0.0107(s_{13}/0.2) \cos \delta)$, while if $s_{23}^2 = 0.36$ one obtains $\tilde{m}_\tau / \tilde{m}_2 \cong 0.5(0.0794 + 0.0077 s_{13}^2 / 0.04 - 0.0494(s_{13}/0.2) \cos \delta)$. The maxima of the asymmetry $|Y_B|$ take place at $\delta = \pi/2 + k\pi$, $k = 0, 1, 2, \dots$. For $\delta = \pi/2$, $s_{13} = 0.2$ and $s_{23}^2 = 0.64$; 0.5 ; 0.36 we get: $\tilde{m}_\tau / \tilde{m}_2 \cong 0.52 \times 10^{-2}$; 0.81×10^{-2} ; 4.36×10^{-2} . Therefore the two maxima of $|Y_B|$ as a function of m_3 , corresponding to the CP-asymmetry being predominantly in the $(e + \mu)$ -flavour and in the τ -flavour, can be expected to occur at values of m_3 , which for $s_{23}^2 = 0.36$; 0.5 ; 0.64 and $s_{12}^2 = 0.30$ would differ by a factor

¹³As is well-known, the Majorana phases, in contrast to the Dirac phase, do not affect the flavour neutrino oscillations [29, 37].

¹⁴If we use $M_1 \lesssim 10^{12} \text{ GeV}$ [15, 16, 17] for the maximal value of M_1 in this discussion, we get, obviously, $|\sin \theta_{13} \sin \delta| \gtrsim 0.044$, $\sin \theta_{13} \gtrsim 0.044$ and $|J_{\text{CP}}| \gtrsim 0.01$.

of $\tilde{m}_2/\tilde{m}_\tau \sim 20; 120; 190$. The position of the deep minimum of $|Y_B|$ between the two maxima would also be very different for $s_{23}^2 = 0.36$ and $s_{23}^2 = 0.5$ (0.64). Obviously, the relative position on the m_3 axis of two maxima and the minimum of $|Y_B|$ under discussion will depend not only on the precise value of $\sin^2 \theta_{23}$, but also on the precise value of $\sin^2 \theta_{12}$.

To be more concrete, the maximum of $|Y_B|$ (as a function of m_3), associated with the CP-asymmetry being predominantly in the $(e + \mu)$ -flavour, takes place at $m_3 \cong 7.5 \times 10^{-4}$ eV, i.e. in the region of IH spectrum. At this value of m_3 , $\eta(0.71\tilde{m}_2) \cong$ maximal, $\eta(0.71\tilde{m}_2) \cong 0.068$, while $\eta(0.66\tilde{m}_\tau) \lesssim 0.005 \ll \eta(0.71\tilde{m}_2)$, and we have:

$$|Y_B| \cong 5.1 \times 10^{-13} |\sin \delta| \frac{\sin \theta_{13}}{0.2} \left(\frac{\sqrt{|\Delta m_A^2|}}{0.05 \text{ eV}} \right) \left(\frac{M_1}{10^9 \text{ GeV}} \right). \quad (36)$$

The position of this maximum does not depend on θ_{12} , θ_{23} , θ_{13} and δ . Thus, the measured value of $|Y_B|$, $8.0 \times 10^{-11} \lesssim |Y_B| \lesssim 9.2 \times 10^{-11}$, can be reproduced for a somewhat smaller value of $M_1 \gtrsim 1.6 \times 10^{11}$ GeV than the corresponding value of M_1 we have found for $\rho_{23} = -1$ (compare eqs. (32) and (36)). In the vicinity of the maximum there exists a correlation between the values of $|Y_B|$ and $|J_{CP}|$ similar to the one given in eq. (35). Now the requirement of successful leptogenesis leads for $M_1 \lesssim 5 \times 10^{11}$ GeV to a somewhat less stringent lower limit on $|\sin \theta_{13} \sin \delta|$, and thus on $\sin \theta_{13}$ and $|J_{CP}|$:

$$|\sin \theta_{13} \sin \delta|, \sin \theta_{13} \gtrsim 0.063, \quad |J_{CP}| \gtrsim 0.015. \quad (37)$$

The second maximum of $|Y_B|$, related to the possibility of the CP-asymmetry being predominantly in the τ -flavour, takes place, as it is not difficult to convince oneself, at $m_2 m_3 / (m_2 + m_3) \gtrsim 10^{-2}$ eV, i.e. for values of $m_3 \gtrsim 1.2 \times 10^{-2}$ eV in the region of neutrino mass spectrum with partial inverted hierarchy. In this case the factor in $|Y_B|$, which determines the position of the maximum as a function of m_3 , is $((m_2 - m_3) / \sqrt{|\Delta m_A^2|}) \eta(0.66\tilde{m}_\tau)$, rather than just $\eta(0.66\tilde{m}_\tau)$. Taking this observation into account, it is not difficult to show that for $\delta = \pi/2$ and $s_{13} = 0.2$ maximising $|Y_B|$, $s_{12}^2 = 0.30$ and, e.g. $s_{23}^2 = 0.36$ (0.50), the maximum occurs at $m_3 \cong 1.8$ (5.0) $\times 10^{-2}$ eV. If $M_1 = 10^{11}$ GeV and $\sqrt{|\Delta m_A^2|} = 5.0 \times 10^{-2}$ eV, the value of $|Y_B|$ at this maximum reads: $|Y_B| \cong 4.4$ (1.1) $\times 10^{-11}$. For $s_{23}^2 = 0.64$ we get for the same values of the other parameters $\max(|Y_B|) \cong 0.6 \times 10^{-11}$. Obviously, if $m_3 \gtrsim 10^{-2}$ eV, the observed value of $|Y_B|$ can be reproduced for $M_1 \lesssim 5 \times 10^{11}$ GeV only if $s_{23}^2 < 0.50$. The position of the deep minimum of $|Y_B|$ at $m_3 \gtrsim 10^{-3}$ eV, as we have already indicated, is also very sensitive to the value of s_{23}^2 : for $\delta = \pi/2$, $s_{13} = 0.2$ and $s_{12}^2 = 0.30$, it takes place at $m_3 \cong 2 \times 10^{-3}$ eV if $s_{23}^2 = 0.36$, and at $m_3 \cong 10^{-2}$ eV in the case of $s_{23}^2 = 0.50$. These features of the dependence of $|Y_B|$ on m_3 are illustrated in Fig. 4.

One can perform a similar analysis in the case of real $R_{1j} R_{1k}$, $R_{12} = 0$ and $R_{11}, R_{13} \neq 0$. In this case we have $|\epsilon_\tau| \propto |\text{Im}(U_{\tau 1}^* U_{\tau 3})| = c_{23}^2 c_{13} c_{12} s_{13} |\sin \delta| \lesssim 0.082 |\sin \delta|$, and

$$\tilde{m}_\tau \cong \frac{m_1 m_3}{m_3 + m_1} \left[(s_{12} s_{23} + \rho_{13} c_{13} c_{23})^2 + c_{12}^2 c_{23}^2 s_{13}^2 - 2 s_{13} c_{12} c_{23} (s_{12} s_{23} + \rho_{13} c_{23} c_{13}) \cos \delta \right], \quad (38)$$

$\tilde{m}_2 = 2m_1m_3/(m_3 + m_1) - \tilde{m}_\tau$, where $\rho_{13} \equiv \text{sgn}(R_{11}R_{13}) = \pm 1$ and $m_1 \cong m_2 = \sqrt{m_3^2 + |\Delta m_A^2|}$. For $\rho_{13} = +1$, the two maxima of $|Y_B|$ (as a function of m_3) have the same magnitude. They occur at $\delta \cong 3\pi/4$, $s_{13} = 0.2$ and $m_3 \cong 7.5 \times 10^{-4}$ (3.5×10^{-3}) eV. The maximal baryon asymmetry, $\max(|Y_B|)$, exhibits rather strong dependence on s_{23}^2 . For $s_{23}^2 = 0.36$ (0.50), $M_1 = 5 \times 10^{11}$ GeV and $\sqrt{|\Delta m_A^2|} = 5.0 \times 10^{-2}$ eV, we get $\max(|Y_B|) \cong 1.7$ (0.9) $\times 10^{-10}$. If $s_{23}^2 > 0.50$, however, it is impossible to reproduce the observed value of $|Y_B|$ for $M_1 \lesssim 5 \times 10^{11}$ GeV. The same negative result holds for any s_{23}^2 from the interval $[0.36 - 0.64]$ if $s_{13} \lesssim 0.10$.

In the case of $\rho_{13} = -1$, we have $|Y_B| \propto c_{23}^2$ in the region of the maximum of $|Y_B|$ at $m_3 \cong 7.5 \times 10^{-4}$ eV, associated with the CP-asymmetry being predominantly in the $(e+\mu)$ -flavour. The baryon asymmetry $|Y_B|$ is maximal for $\delta = \pi/2$ which maximises the CP-asymmetry $|\epsilon_\tau|$. For $s_{13} = 0.2$, $c_{23}^2 = 0.5$, $M_1 = 5 \times 10^{11}$ GeV and $\sqrt{|\Delta m_A^2|} = 5.0 \times 10^{-2}$ eV, we find $\max(|Y_B|) \cong 4.5 \times 10^{-10}$:

$$|Y_B| \cong 9.0 \times 10^{-13} |\sin \delta| \frac{\sin \theta_{13}}{0.2} \left(\frac{\sqrt{|\Delta m_A^2|}}{0.05 \text{ eV}} \right) \left(\frac{M_1}{10^9 \text{ GeV}} \right). \quad (39)$$

Thus, the observed value of the baryon asymmetry can be reproduced for relatively small values of $|\sin \theta_{13} \sin \delta|$, and correspondingly of $\sin \theta_{13}$ and $|J_{\text{CP}}|$:

$$|\sin \theta_{13} \sin \delta|, \sin \theta_{13} \gtrsim 0.036, \quad |J_{\text{CP}}| \gtrsim 0.0086. \quad (40)$$

In contrast, the position (with respect to m_3) of the maximum of $|Y_B|$, associated with the CP-asymmetry being predominantly in the τ -flavour, and its magnitude, exhibit rather strong dependence on s_{23}^2 . For $s_{23}^2 = 0.36; 0.50; 0.64$, the maximum of $|Y_B|$ is located at $m_3 \cong 0.7; 1.5; 3.0 \times 10^{-2}$ eV. For $M_1 \lesssim 5 \times 10^{11}$ GeV, the measured value of $|Y_B|$, $8.0 \times 10^{-11} \lesssim |Y_B| \lesssim 9.2 \times 10^{-11}$, can be reproduced provided $|\sin \theta_{13} \sin \delta| \gtrsim 0.046; 0.053; 0.16$ if $s_{23}^2 = 0.36; 0.50; 0.64$.

3.2 Neutrino Mass Spectrum with Normal Ordering

We can get different results for light neutrino mass spectrum with normal ordering. The case of negligible m_1 and real (CP-conserving) elements R_{1j} of R was analysed in detail in [9]¹⁵. It was found that if the only source of CP-violation is the Dirac phase δ in the PMNS matrix, the observed value of the baryon asymmetry $Y_B \cong 8.6 \times 10^{-11}$ can be reproduced if [9] $|\sin \theta_{13} \sin \delta| \gtrsim 0.09$. Given the upper limit $|\sin \theta_{13} \sin \delta| < 0.2$, this requires $M_1 \gtrsim 2 \times 10^{11}$ GeV. The quoted lower limit on $|\sin \theta_{13} \sin \delta|$ implies that we should have $\sin \theta_{13} \gtrsim 0.09$ and that $|J_{\text{CP}}| \gtrsim 2 \times 10^{-2}$. The indicated values of $\sin \theta_{13}$ and of $|J_{\text{CP}}|$ are testable in the reactor neutrino and long baseline neutrino oscillation experiments under preparation [39, 40, 41, 42]. If, however, the Dirac phase δ has a CP-conserving value, $\delta \cong k\pi$, $k = 0, 1, 2, \dots$, and the requisite CP violation is due exclusively

¹⁵Results for quasi-degenerate (QD) spectrum, $m_1 \cong m_2 \cong m_3$, $m_j^2 \gg |\Delta m_A^2|$, which requires $m_j \gtrsim 0.10$ eV, were also obtained in [9].

to the Majorana phases $\alpha_{21,31}$ in U , the observed Y_B can be obtained for [9] $M_1 \gtrsim 4 \times 10^{10}$ GeV. For $M_1 = 5 \times 10^{11}$ GeV, for which the flavour effects are fully developed, the measured value of Y_B can be reproduced for a rather small value of $|\sin \alpha_{32}/2| \cong 0.15$, where $\alpha_{32} \equiv \alpha_{31} - \alpha_{21}$.

In searching for possible significant effects of non-negligible m_1 in leptogenesis we have considered values of m_1 as large as 0.05 eV, $m_1 \leq 0.05$ eV. For 3×10^{-3} eV $\lesssim m_1 \lesssim 0.10$ eV, the neutrino mass spectrum is not hierarchical; the spectrum exhibits partial hierarchy (see, e.g. [28]), i.e. we have $m_1 < m_2 < m_3$.

We are interested in the dependence of the baryon asymmetry on the value of m_1 . In the case of neutrino mass spectrum with normal ordering we have:

$$m_2 = \sqrt{m_1^2 + \Delta m_\odot^2}, \quad m_3 = \sqrt{m_1^2 + \Delta m_A^2}. \quad (41)$$

We will illustrate the characteristic features of the possible effects of m_1 in leptogenesis by analysing two simple possibilities: $|R_{11}| \ll 1$ and $|R_{12}| \ll 1$. Results of a more general analysis performed without making a priori assumptions about the real parameters R_{11} and R_{12} will also be presented.

We first set $R_{11} = 0$. The asymmetry ϵ_τ is given by:

$$\epsilon_\tau \cong -\frac{3M_1\sqrt{\Delta m_A^2}}{16\pi v^2} \left(\frac{m_3}{m_2}\right)^{\frac{1}{2}} \frac{\sqrt{\Delta m_A^2}}{m_2 + m_3} \rho_{23} r \operatorname{Im}(U_{\tau 2}^* U_{\tau 3}), \quad (42)$$

where

$$r = \frac{|R_{12}R_{13}|}{|R_{12}|^2 + \frac{m_3}{m_2}|R_{13}|^2}, \quad (43)$$

and $\operatorname{Im}(U_{\tau 2}^* U_{\tau 3})$ is given in eq. (18). The ratio in (43) is similar to the ratio in eq. (17). Note, however, that the masses $m_{2,3}$ present in eqs. (15) and (17) are very different from the masses $m_{2,3}$ in eqs. (42) and (43). Using again the fact that $R_{12}^2 + R_{13}^2 = 1$ and the results given in eq. (21), it is easy to find that r has a maximum for

$$R_{12}^2 = \frac{m_3}{m_2 + m_3}, \quad R_{13}^2 = \frac{m_2}{m_2 + m_3}, \quad R_{13}^2 < R_{12}^2, \quad (44)$$

where m_2 and m_3 are defined now by eq. (41). At the maximum

$$\max(r) = \frac{1}{2} \left(\frac{m_2}{m_3}\right)^{\frac{1}{2}}. \quad (45)$$

For the value of R_{12} (R_{13}), which maximises the ratio $|r|$ and correspondingly the asymmetry $|\epsilon_\tau|$, the relevant wash-out mass parameters \tilde{m}_τ and \tilde{m}_2 are given by eqs. (27) and (28) with m_2 and m_3 determined by eq. (41). Since in the case of interest $m_2 \gtrsim \sqrt{\Delta m_\odot^2} \cong 0.9 \times 10^{-2}$ eV, $m_3 \gtrsim \sqrt{\Delta m_A^2} \cong 5.0 \times 10^{-2}$ eV, we have $m_2 m_3 / (m_2 + m_3) \gtrsim 0.7 \times 10^{-2}$ eV. The lightest neutrino mass m_1 can have any effect on the generation of the baryon

asymmetry Y_B only if $m_1^2 \gg \Delta m_\odot^2$ and if m_1 is non-negligible with respect to $\sqrt{\Delta m_A^2}$. Correspondingly, for non-negligible m_1 of interest we will have $m_2 m_3 / (m_2 + m_3) \gtrsim 10^{-2}$ eV and the baryon asymmetry will be generated in the "strong wash-out" regime, unless there is a strong cancellation between the first two and the third terms in the expression for \tilde{m}_τ , see eq. (27). Obviously, the possibility of such a cancellation depends critically on the $\text{sgn}(R_{12}R_{13}) = \rho_{23}$. It should also be clear from eq. (41) and the dependence on $m_{2,3}$ of $\max(|\epsilon_\tau|)$ and of the corresponding \tilde{m}_τ and \tilde{m}_2 (see eqs. (42), (45), (27) and (28)) that with the increasing of m_1 beyond $\sim 10^{-2}$ eV the predicted baryon asymmetry decreases.

A. Leptogenesis due to Majorana CP-Violation in U_{PMNS}

Suppose first that the Dirac phase δ in the PMNS matrix has a CP-conserving value, $\delta = \pi k$, $k = 0, 1, 2, \dots$, and that the only source of CP-violation are the Majorana phases $\alpha_{21,31}$ in the PMNS matrix U . In the specific case of $R_{11} = 0$ we are considering the relevant CP-violating parameter is the difference of the two Majorana phases $\alpha_{32} \equiv \alpha_{31} - \alpha_{21}$. In this case $|\epsilon_\tau| \propto \text{Im}(U_{\tau 2}^* U_{\tau 3}) \cong c_{23}^2 c_{12} |\sin \alpha_{32}/2| \cong 0.42 |\sin \alpha_{32}/2|$, where we have neglected the possible subleading corrections due to terms proportional to $\sin \theta_{13}$. Even for $\sin \theta_{13} = 0.2$ the latter have practically no influence on the results we are going to obtain and for simplicity we set $\sin \theta_{13} = 0$ in the discussion of Majorana CP-violation which follows. For the wash-out mass parameter \tilde{m}_τ we find:

$$\tilde{m}_\tau \cong m_2 \frac{m_3}{m_2 + m_3} \left[c_{12}^2 s_{23}^2 + c_{23}^2 - 2 \rho_{23} c_{23} s_{23} c_{12} \cos \frac{\alpha_{32}}{2} \right]. \quad (46)$$

If $\cos \alpha_{32}/2 \cong 0$, the baryon asymmetry Y_B is produced in the strong wash-out regime and for $M_1 < 10^{12}$ GeV it is impossible to reproduce the observed value of $Y_B \cong 8.6 \times 10^{-11}$: the calculated asymmetry is too small. Actually, the maximum of $|Y_B|$ in the case under discussion occurs, as can be shown, for $\alpha_{32} \cong \pi/2 + \pi k$, $k = 0, 1, 2, \dots$. There are two distinctive possibilities to be considered corresponding to the two possible signs of $\rho_{23} \text{sgn}(\cos \alpha_{32}/2)$. If $\rho_{23} \text{sgn}(\cos \alpha_{32}/2) = +1$, we have $\tilde{m}_\tau \cong 0.25 m_2 m_3 / (m_2 + m_3)$, the asymmetry in the τ -flavour ($(e + \mu)$ -flavour) is produced in the weak (strong) wash-out regime and we get for, e.g. $m_1 = 2 \times 10^{-2}$ (5×10^{-2}) eV:

$$|Y_B| \cong 1.20 (0.36) \times 10^{-12} \left(\frac{\sqrt{\Delta m_A^2}}{0.05 \text{ eV}} \right) \left(\frac{M_1}{10^9 \text{ GeV}} \right), \quad \alpha_{32} \cong \pi/2 + \pi k. \quad (47)$$

Thus, for $m_1 = 2 \times 10^{-2}$ (5×10^{-2}) eV the measured value of $Y_B \cong 8.6 \times 10^{-11}$ can be obtained for $M_1 \gtrsim 7.2 \times 10^{10}$ (2.4×10^{11}) GeV.

These results are illustrated in Fig. 5, showing the correlated values of M_1 and m_1 for which one can have successful leptogenesis. The figure was obtained using the same general method of analysis we have employed to produce Fig. 1. For given m_1 from the interval $10^{-10} \leq m_1 \leq 0.05$ eV, a thorough scan of the relevant parameter space was performed in the calculation of $|Y_B|$, searching for possible non-standard features (enhancement or suppression) of the baryon asymmetry. The real elements R_{1j} of interest

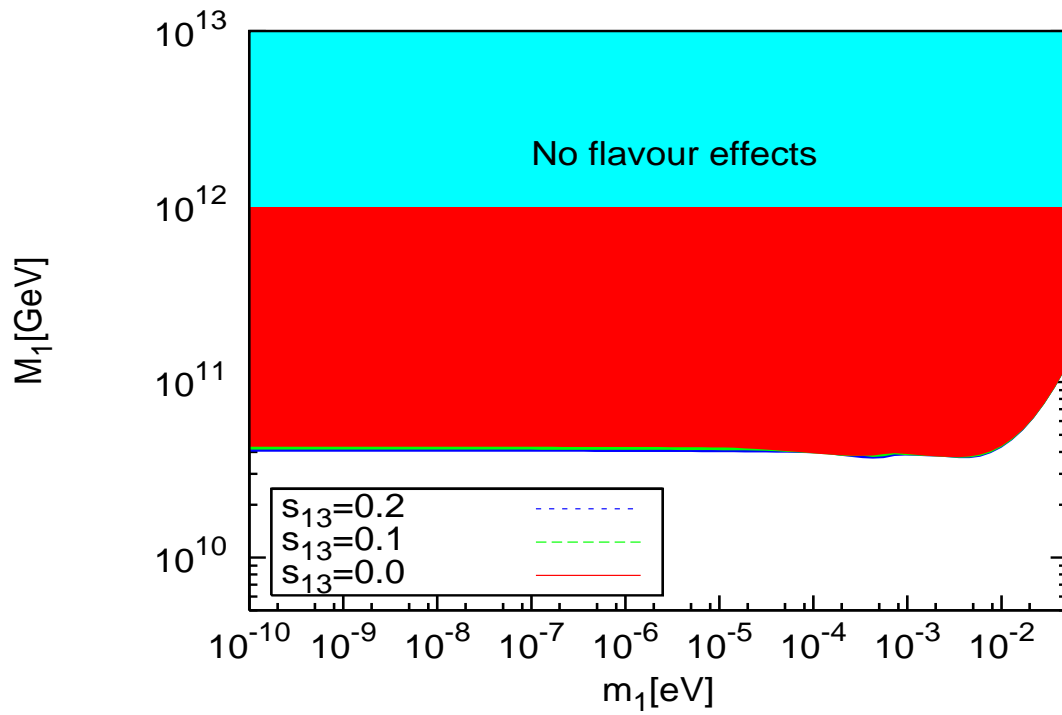


Figure 5: Values of m_1 and M_1 for which the “flavoured” leptogenesis is successful and baryon asymmetry $Y_B = 8.6 \times 10^{-11}$ can be generated (red shaded area). The figure corresponds to light neutrino mass spectrum with normal ordering. The CP-violation necessary for leptogenesis is due to the Majorana and Dirac phases in the PMNS matrix. The results shown are obtained using the best fit values of neutrino oscillation parameters: $\Delta m_{\odot}^2 = 8.0 \times 10^{-5} \text{ eV}^2$, $\Delta m_{\text{A}}^2 = 2.5 \times 10^{-3} \text{ eV}^2$, $\sin^2 \theta_{12} = 0.30$ and $\sin^2 2\theta_{23} = 1$. See text for further details.

of the matrix R , were allowed to vary in their full ranges determined by the condition of orthogonality of R : $R_{11}^2 + R_{12}^2 + R_{13}^2 = 1$. The Majorana and Dirac phases $\alpha_{21,31}$ and δ were varied in the interval $[0, 2\pi]$. The calculations were performed again for three values of the CHOOZ angle, $\sin \theta_{13} = 0; 0.1; 0.2$. The relevant heavy Majorana neutrino mass M_1 was varied in the interval $10^9 \text{ GeV} \lesssim M_1 \lesssim 10^{12} \text{ GeV}$. For given m_1 , the minimal value of the mass M_1 , for which the leptogenesis is successful generating $Y_B \cong 8.6 \times 10^{-11}$, was obtained for the values of the other parameters which maximise $|Y_B|$. The $\min(M_1)$ thus calculated did not show any significant dependence on s_{13} . For $m_1 \lesssim 7.5 \times 10^{-3} \text{ eV}$ we did not find any noticeable effect of m_1 in leptogenesis: the results we have obtained practically coincide with those corresponding to $m_1 = 0$ and derived in [9]. The minimal value of $M_1 \cong 4 \times 10^{10} \text{ GeV}$ seen in Fig. 1 corresponds to $R_{12}^2 \cong 0.85$, $R_{13}^2 \cong 0.15$ and $\alpha_{32} \cong \pi/2$ ($\rho_{23} \text{sgn}(\cos \alpha_{32}/2) = +1$); it does not depend on m_1 . For $7.5 \times 10^{-3} \text{ eV} \lesssim m_1 \leq 5 \times 10^{-2} \text{ eV}$, as our calculations and Fig. 1 show, the predicted baryon asymmetry Y_B for given M_1 is generically smaller with respect to the asymmetry Y_B one finds for $m_1 = 0$. Thus, successful leptogenesis is possible for larger values of $\min(M_1)$. The corresponding suppression factor increases with m_1 and for $m_1 \cong 5 \times 10^{-2} \text{ eV}$ values of $M_1 \gtrsim 10^{11} \text{ GeV}$ are required.

If, however, $\rho_{23}\text{sgn}(\cos \alpha_{32}/2) = -1$, both the asymmetries in the τ -flavour and in the $(e + \mu)$ -flavour are generated under the conditions of strong wash-out effects. Correspondingly, it is impossible to have a successful leptogenesis for $M_1 < 10^{12}$ GeV if $m_1 \cong 5 \times 10^{-2}$ eV. If m_1 has a somewhat lower value, say $m_1 = 2 \times 10^{-2}$ eV, the wash-out of the $(e + \mu)$ -flavour asymmetry is less severe ($\tilde{m}_2 \cong 8.6 \times 10^{-3}$ eV) and the observed Y_B can be reproduced for $\alpha_{32} = \pi/2 + \pi k$ if $M_1 \gtrsim 2.5 \times 10^{11}$ GeV.

B. Leptogenesis due to Dirac CP-Violation in U_{PMNS}

If the Majorana phases $\alpha_{21,31}$ have CP-conserving values and the only source of CP-violation is the Dirac phase δ in U_{PMNS} , one has $|\epsilon_\tau| \propto |c_{23}^2 c_{13} s_{12} s_{13} \sin \delta| \lesssim 0.054 |\sin \delta|$. The factor $c_{23}^2 c_{13} s_{12} s_{13}$ in $|\epsilon_\tau|$ is smaller by at least approximately an order of magnitude than the analogous factor $c_{23}^2 c_{12}$ in $|\epsilon_\tau|$ corresponding to Majorana CP-violation we have considered above. The study of this case of Dirac CP-violation we have performed shows that as a consequence of the indicated suppression it is impossible to generate the observed value of the baryon asymmetry $|Y_B|$ for $M_1 \lesssim 5 \times 10^{11}$ GeV.

We have investigated also the possibility of $|R_{13}|$ being sufficiently small, so that the term $\propto R_{11}R_{12}$ in the expression for ϵ_τ is the dominant one. We have found that if $R_{13} = 0$, it is impossible to have successful leptogenesis for $m_1 \lesssim 0.05$ eV and $M_1 < 10^{12}$ GeV if the requisite CP-violation is due to the Majorana and/or Dirac phases in U_{PMNS} . The same conclusion is valid for normal hierarchical spectrum, $m_1 \ll m_2 \cong \sqrt{\Delta m_\odot^2}$.

C. The case of $R_{12} = 0$

We get very different results if $R_{12} = 0$, while $R_{11}R_{13} \neq 0$. In this case the expression for the asymmetry ϵ_τ can be obtained formally from eq. (42) by replacing m_2 with m_1 , ρ_{23} with ρ_{13} , $U_{\tau 2}^*$ with $U_{\tau 1}^*$ and the ratio r with

$$r = \frac{|R_{11} R_{13}|}{|R_{11}|^2 + \frac{m_3}{m_1} |R_{13}|^2} \ , \quad R_{11}^2 + R_{13}^2 = 1 \ . \quad (48)$$

As in the similar cases discussed earlier, the ratio r , and the asymmetry $|\epsilon_\tau|$, take maximal values for

$$R_{11}^2 = \frac{m_3}{m_1 + m_3} \ , \quad R_{13}^2 = \frac{m_1}{m_1 + m_3} \ , \quad (49)$$

and $\max(r) = 0.5(m_1/m_3)^{\frac{1}{2}}$. The expression of the asymmetry $|\epsilon_\tau|$ at the maximum (with respect to R_{11}) reads:

$$|\epsilon_\tau| \cong \frac{3M_1 \sqrt{\Delta m_A^2}}{32\pi v^2} \frac{\sqrt{\Delta m_A^2}}{m_1 + m_3} |\text{Im}(U_{\tau 1}^* U_{\tau 3})| \ . \quad (50)$$

The wash-out parameters \tilde{m}_τ and \tilde{m}_2 , corresponding to the maximum $|\epsilon_\tau|$, are given by

$$\tilde{m}_\tau = \frac{m_1 m_3}{m_1 + m_3} \left[|U_{\tau 1}|^2 + |U_{\tau 3}|^2 + 2 \rho_{13} \text{Re}(U_{\tau 1}^* U_{\tau 3}) \right] \quad (51)$$

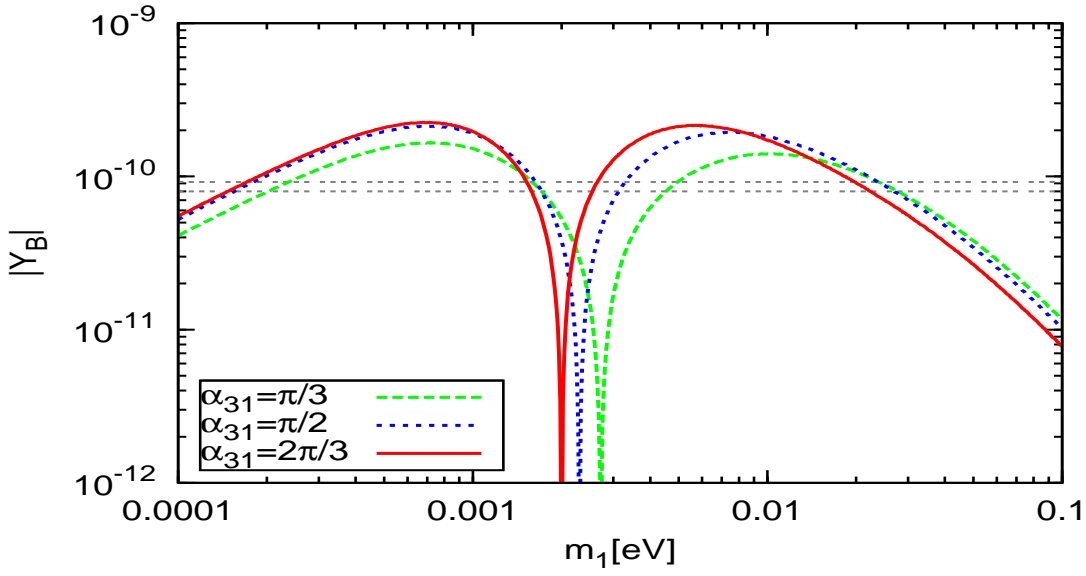


Figure 6: The dependence of $|Y_B|$ on m_1 in the case of neutrino mass spectrum with normal ordering and real $R_{1j}R_{1k}$, for $R_{12} = 0$, $s_{13} = 0$, $M_1 = 1.5 \times 10^{11}$ GeV and $\text{sgn}(R_{11}R_{13}) = -1$. The red solid, the blue dotted, and the green dashed lines correspond to $\alpha_{31} = 2\pi/3$, $\pi/2$, and $\pi/3$ respectively. The figure is obtained for $\theta_{23} = \pi/4$.

$$= \frac{m_1 m_3}{m_1 + m_3} \left(s_{12}^2 s_{23}^2 + c_{23}^2 + 2 \rho_{13} c_{23} s_{23} s_{12} \cos \frac{\alpha_{31}}{2} \right), \quad (52)$$

where we have set $s_{13} = 0$ in obtaining the second expression, and $\tilde{m}_2 = 2m_1 m_3 / (m_1 + m_3) - \tilde{m}_\tau$. Note that if $m_1 \ll m_3 \cong 5 \times 10^{-2}$ eV, the asymmetry $|\epsilon_\tau|$ practically does not depend on m_1 , while $\tilde{m}_{\tau,2} \sim \mathcal{O}(m_1)$. This implies that the dependence of $\max(|Y_B|)$ on m_1 as the latter increases, will exhibit the same features as in the case of IH spectrum we have already discussed: $|Y_B|$ will have two maxima, corresponding to the CP-asymmetry being predominantly in the τ -flavour and in the $(e + \mu)$ -flavour, separated with a deep minimum. The analysis of the similar case of IH spectrum suggests, that for $s_{13} = 0$, the largest baryon asymmetry $|Y_B|$ is obtained for $\alpha_{31} \neq \pi(2k+1)$ and $\rho_{13} \text{sgn}(\cos \alpha_{31}/2) = -1$. These features are confirmed by our numerical calculations and are illustrated in Fig. 6.

In Fig. 6 we have displayed the dependence of $|Y_B|$ on m_1 in the case of neutrino mass spectrum with normal ordering, $R_{12} = 0$ and real $R_{11}R_{13}$. The results shown are obtained for $\rho_{13} = -1$, $\sin \theta_{13} = 0$, $M_1 = 3 \times 10^{11}$ GeV, and three CP-violating values of the Majorana phase α_{31} , relevant for the calculation of $|Y_B|$: $2\pi/3$; $\pi/2$; $\pi/3$. The two maxima and the deep minimum of $|Y_B|$ are evident in the figure. The maximal values of $|Y_B|$ are reached for $\alpha_{31} \cong 2\pi/3$. Actually, in what concerns the dependence of $|Y_B|$ on α_{31} and ρ_{13} in the case of $s_{13} = 0$, the following relation holds: $|Y_B(\rho_{13}, \alpha_{31})| = |Y_B(-\rho_{13}, 2\pi - \alpha_{31})|$. The two maxima in question occur at $m_1 \cong 7.7 \times 10^{-4}$ eV and at $m_1 \cong 5.5 \times 10^{-3}$ eV. At these maximum points we have $\eta(0.66\tilde{m}_\tau) - \eta(0.71\tilde{m}_2) \cong (-0.044)$ and $(+0.047)$, respectively. The complete compensation between $\eta(0.66\tilde{m}_\tau)$ and $\eta(0.71\tilde{m}_2)$, leading to $|Y_B| \cong 0$, takes place at $m_1 \sim 1.5 \times 10^{-3}$ eV. For $\alpha_{31} = 2\pi/3$, the baryon asymmetry at

the two maxima reads:

$$|Y_B| \cong 1.5 \text{ (1.4)} \times 10^{-12} \left(\frac{\sqrt{|\Delta m_A^2|}}{0.05 \text{ eV}} \right) \left(\frac{M_1}{10^9 \text{ GeV}} \right). \quad (53)$$

Thus, one can have successful leptogenesis for $M_1 \gtrsim 5.3 \times 10^{10} \text{ GeV}$.

Similar analysis can be performed assuming that the only source of CP-violation in leptogenesis is the Dirac phase δ in the neutrino mixing matrix. The results one obtains in this case are analogous to those derived in the last part of Section 3.1 (see eqs. (38), (39) and (40) and the related discussion).

4 Conclusions

In the present article we have investigated the dependence of the “flavoured” (thermal) leptogenesis on the lightest neutrino mass in the case when the CP-violation necessary for the generation of the observed baryon asymmetry of the Universe is due exclusively to the Majorana and/or Dirac CP-violating phases in the PMNS neutrino mixing matrix U_{PMNS} . The two possible types of light neutrino mass spectrum allowed by the data were considered: i) with normal ordering ($\Delta m_A^2 > 0$), $m_1 < m_2 < m_3$, and ii) with inverted ordering ($\Delta m_A^2 < 0$), $m_3 < m_1 < m_2$. The study was performed within the simplest type I see-saw scenario with three heavy Majorana neutrinos N_j , $j = 1, 2, 3$, having a hierarchical mass spectrum with masses $M_1 \ll M_{2,3}$. Throughout this analysis we used the “orthogonal” parametrisation of the matrix of neutrino Yukawa couplings, involving an orthogonal matrix R , $R^T R = R R^T = \mathbf{1}$. The latter, in general, can be complex, i.e. CP-violating. In the present work we were primarily interested in the possibility that R is real, conserves the CP-symmetry, and the violation of CP-symmetry necessary for leptogenesis is due exclusively to the CP-violating phases in U_{PMNS} . In the case of hierarchical heavy Majorana neutrinos $N_{1,2,3}$, $M_1 \ll M_2 \ll M_3$, the CP-violating lepton charge (flavour) asymmetries ϵ_l , $l = e, \mu, \tau$, relevant in leptogenesis, are produced in the decays of the lightest one, N_1 . As a consequence, the generated baryon asymmetry Y_B depends (linearly) on the mass of N_1 , M_1 , and on the elements R_{1j} of the matrix R , $j = 1, 2, 3$, present in the neutrino Yukawa couplings of N_1 .

Our analysis was performed under the condition of negligible RG running from M_Z to M_1 of m_j and of the parameters in U_{PMNS} . This condition is satisfied for sufficiently small values of the lightest neutrino mass, $\min(m_j) \lesssim 0.10 \text{ eV}$. The latter requirement is fulfilled for the normal hierarchical (NH), $m_1 \ll m_2 < m_3$, and inverted hierarchical (IH), $m_3 \ll m_1 < m_2$, spectra, and for the spectra with partial hierarchy. Therefore the neutrino masses m_j , the solar, atmospheric and CHOOZ mixing angles, θ_{12} , θ_{23} and θ_{13} , and the Majorana and Dirac CP-violating phases, $\alpha_{21,31}$ and δ , present in U_{PMNS} , are taken at the scale $\sim M_Z$, at which the neutrino mixing parameters are measured.

We have investigated in detail the case of light neutrino mass spectrum with inverted ordering and real (and CP-conserving) matrix R , considering values of the lightest neu-

trino mass m_3 in the range $10^{-10} \text{ eV} \leq m_3 \leq 0.05 \text{ eV}$. The case of IH spectrum, $m_3 \ll m_1 < m_2$, $m_{1,2} \cong \sqrt{|\Delta m_A^2|}$, is of major interest since, for real elements R_{1j} of R , IH spectrum, $m_3 \cong 0$ and CP-violation due to the Majorana and/or Dirac phases in U_{PMNS} , it was shown [9] to be impossible to generate the observed baryon asymmetry $Y_B \cong 8.6 \times 10^{-11}$ in the regime of “flavoured” leptogenesis (i.e. for $M_1 \lesssim 10^{12} \text{ GeV}$): the resulting baryon asymmetry is too small. Therefore our investigation was focused primarily on the effects in leptogenesis of a non-negligible m_3 , having a value in the range of the IH spectrum: $10^{-6} \text{ eV} \lesssim m_3 \lesssim 5 \times 10^{-3} \text{ eV}$, $m_3 \ll m_{1,2} \cong \sqrt{|\Delta m_A^2|} \cong 5 \times 10^{-2} \text{ eV}$. These effects can be particularly large if $R_{11} \cong 0$ or $R_{12} \cong 0$.

We have found that in the case of IH spectrum with non-negligible m_3 , $m_3 \ll \sqrt{|\Delta m_A^2|}$, the generated baryon asymmetry $|Y_B|$ can be strongly enhanced in comparison with the asymmetry $|Y_B|$ produced if $m_3 \cong 0$. The enhancement can be by a factor of ~ 100 , or even by a larger factor. As a consequence, one can have successful leptogenesis for IH spectrum with $m_3 \gtrsim 5 \times 10^{-6} \text{ eV}$ even if the elements R_{1j} of R are real and the requisite CP-violation is provided by the Majorana or Dirac phase(s) in the PMNS matrix. The dependence of $|Y_B|$ on m_3 has the following characteristic features. As m_3 increases from the value of 10^{-10} eV up to 10^{-4} eV , the maximal possible $|Y_B|$ for a given M_1 increases monotonically, starting from a value which for $M_1 \leq 10^{12} \text{ GeV}$ is much smaller than the observed one, $\max(|Y_B|) \ll 8.6 \times 10^{-11}$. At $m_3 \cong \text{few} \times 10^{-6} \text{ eV}$, we have $\max(|Y_B|) \cong 8.6 \times 10^{-11}$ for $M_1 \cong 5 \times 10^{11} \text{ GeV}$. As m_3 increases beyond $\text{few} \times 10^{-6} \text{ eV}$, $\max(|Y_B|)$ for a given M_1 continues to increase until it reaches a maximum (Figs. 1, 2 and 4). This maximum is located typically at $m_3 \cong (7.0 - 7.5) \times 10^{-4} \text{ eV}$. It corresponds to the CP-asymmetry being predominantly in the $(e + \mu)$ -flavour. As m_3 increases further, $|Y_B|$ rapidly decreases. At certain value of m_3 , typically lying in the interval $m_3 \sim (1.5 - 2.5) \times 10^{-3} \text{ eV}$ or $m_3 \sim (2.0 - 10.0) \times 10^{-3} \text{ eV}$, depending on whether the CP-violation in leptogenesis is caused by the Majorana or Dirac phases in U_{PMNS} , $|Y_B|$ goes through a deep minimum: one can have even $|Y_B| = 0$ (Figs. 1, 2 and 4). This minimum of $|Y_B|$ corresponds to a partial or complete cancellation between the asymmetries in the τ -flavour and in the $(e + \mu)$ -flavour. The position of the minimum in the case of Dirac CP-violation from U_{PMNS} is very sensitive to the precise value of the atmospheric and solar neutrino mixing angles θ_{23} and θ_{12} . The dependence on $\sin^2 \theta_{23}$ is particularly strong (Fig. 4). As m_3 increases further, $|Y_B|$ rapidly increases reaching a second maximum. In magnitude the latter is of the order of the first one in the case of Majorana CP-violation from U_{PMNS} . If the CP-violation is due to the Dirac phase in U_{PMNS} , the second maximum of $|Y_B|$ can be of the order of the first one, or can be significantly smaller, depending on the precise value of $\sin^2 \theta_{23}$ (Fig. 4). This maximum corresponds to the CP-asymmetry being predominantly in the τ -flavour, rather than in the $(e + \mu)$ -flavour. For Majorana (Dirac) CP-violation from U_{PMNS} , the second maximum of $|Y_B|$ occurs typically in the interval $m_3 \cong (4.0 - 8.0) \times 10^{-3} \text{ eV}$ ($m_3 \cong (0.7 - 7.0) \times 10^{-2} \text{ eV}$). As m_3 increases further, $|Y_B|$ decreases rather slowly monotonically. If CP-symmetry is violated by the Majorana phases in U_{PMNS} , we can have successful leptogenesis for $M_1 \gtrsim 3.0 \times 10^{10} \text{ GeV}$. A somewhat larger values of M_1 are typically required if the CP-violation is due to the Dirac phase δ : $M_1 \gtrsim 10^{11} \text{ GeV}$. The requirement of successful “flavoured” leptogenesis in the latter

case leads to the following lower limits on $|\sin \theta_{13} \sin \delta|$, and thus on $\sin \theta_{13}$ and on the rephasing invariant J_{CP} which controls the magnitude of CP-violation effects in neutrino oscillations: $|\sin \theta_{13} \sin \delta|, \sin \theta_{13} \gtrsim (0.04-0.09)$, $|J_{\text{CP}}| \gtrsim (0.009-0.020)$, where the precise value of the limit within the intervals given depends on the $\text{sgn}(R_{11}R_{13})$ (or $\text{sgn}(R_{12}R_{13})$) and on $\sin^2 \theta_{23}$.

The results we have obtained for light neutrino mass spectrum with normal ordering, $m_1 < m_2 < m_3$, depend on whether $R_{11} \cong 0$ or $R_{12} \cong 0$. If $R_{11} \cong 0$, we did not find any significant enhancement of the baryon asymmetry $|Y_B|$, generated within “flavoured” leptogenesis scenario with real matrix R and CP-violation provided by the neutrino mixing matrix U_{PMNS} , when the lightest neutrino mass was varied in the interval $10^{-10} \text{ eV} \leq m_1 \leq 0.05 \text{ eV}$: for $m_1 \lesssim 7.5 \times 10^{-3} \text{ eV}$, the produced asymmetry $|Y_B|$ practically coincides with that corresponding to $m_1 = 0$ (Fig. 5). For $m_1 \gtrsim 10^{-2} \text{ eV}$, the lightest neutrino mass m_1 has a suppressing effect on the asymmetry $|Y_B|$ (Fig. 5). If, however, $R_{12} \cong 0$, the dependence of $|Y_B|$ on m_1 exhibits qualitatively the same features as the dependence of $|Y_B|$ on m_3 in the case of neutrino mass spectrum with inverted ordering (hierarchy) we have summarised above: $|Y_B|$ possesses two maxima separated by a deep minimum (Fig. 6). Quantitatively, $\max(|Y_B|)$ is somewhat smaller than in the corresponding IH spectrum cases. As a consequence, it is possible to reproduce the observed value of Y_B if the CP-violation is due to the Majorana phase(s) in U_{PMNS} provided $M_1 \gtrsim 5.3 \times 10^{10} \text{ GeV}$.

The results obtained in the present article show that the value of the lightest neutrino mass in the cases of neutrino mass spectrum with inverted and normal ordering (hierarchy) can have dramatic effect on the magnitude of the baryon asymmetry of the Universe, generated within the “flavoured” leptogenesis scenario with hierarchical heavy Majorana neutrinos.

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